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Mean lag in general error correction models

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ABSTRACT

Most of the empirical literature inappropriately applies Hendry's (1995) mean lag formula – which he derived for first order autoregressive distributed lag models under the assumption of a homogeneous long-run equilibrium – to error correction models that have complex lag structures and lack long-run homogeneity. We derive an expression for the mean lag in general error correction models without imposing the assumption of a homogeneous equilibrium. In addition, we quantify the bias due to the incorrect use of Hendry's (1995) formula.

1. Introduction

The mean lag is a summary measure of the lag structure of dynamic models. It can be used to estimate the average delay in the transmission of shocks, such as the passthrough of income shocks to consumption, oil price shocks to gas prices, or market interest rates to retail rates, among others. A large number of empirical studies have resorted to an explicit mean lag formula published by Hendry (1995, p. 215, Eq. (6.53)). He derived it for the first order autoregressive distributed lag ($ADL(1, 1)$) model and the associated error correction (EC) model under the assumption of a homogeneous equilibrium relationship. However, the formula is invalid in cases when the lag structure is more complex or the long-run homogeneity assumption does not hold. Nonetheless, we found a number of studies that use the formula inappropriately under these more general conditions, including Chong and Liu (2009); De Bondt (2005); Charoenseang and Manakit (2007); De Graeve et al. (2007); Leibrecht and Scharler (2008, 2011); Leszkiewicz-Kedzior and Welfe (2014); Scholnick (1996), among others.

We fill a gap in the literature by deriving an expression for the mean lag in general EC models without imposing the assumption of a homogeneous equilibrium. In addition, we evaluate the bias of the mean lag estimate arising from inappropriately imposing long-run homogeneity.

2. General form of the mean lag

In this section we derive the mean lag in a general, non-homogeneous, relationship. A general autoregressive distributed lag, or $ADL(p, q; n)$, model can be written as

$$y_t = c + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{k=1}^{n} \sum_{j=0}^{q} \beta_{k,j} x_{k,t-j} + \epsilon_t \quad \text{or}$$
$$\alpha(L) y_t = c + \sum_{k=1}^{n} \beta_k(L) x_{k,t} + \epsilon_t, \tag{1}$$

where $\epsilon_t \sim IID$, $\alpha(L) = 1 - \sum_{i=1}^{p} \alpha_i L^i$ and $\beta_k(L) = \sum_{j=0}^{q} \beta_{k,j} L^j$ are lag polynomials, and n is the number of exogenous variables in the model. Regressors with varying lag lengths can be readily accommodated at the cost of further notational complexity. Rearrange Eq. (1) to obtain the reduced form equation

$$y_t = \frac{c}{\alpha(L)} + \frac{1}{\alpha(L)} \sum_{k=1}^{n} \beta_k(L) x_{k,t} + \frac{\epsilon_t}{\alpha(L)}$$
$$= c^* + \sum_{k=1}^{n} w_k(L) x_{k,t} + u_t, \tag{2}$$

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where $w_k(L) = \frac{\beta_k(L)}{\alpha(L)} = \sum_{j=1}^{\infty} w_{k,j}L^j$. The "weight" associated with lag j of variable x_k, $w_{k,j} = \frac{\partial y_t}{\partial x_{k,t-j}}$, captures the effect of $x_{k,t-j}$ on y_t. Hendry (1995, p. 215) defined the mean lag as

$$\mu_k = \frac{\sum_{j=0}^{\infty} j w_{k,j}}{\sum_{j=0}^{\infty} w_{k,j}} = \frac{1}{w_k(1)}\left[\frac{\partial w_k(L)}{\partial L}\right]_{L=1}$$
$$= \frac{1}{w_k(1)}\left[\frac{\beta_k'(L)}{\alpha(L)} - \frac{\beta_k(L)\alpha'(L)}{\alpha(L)^2}\right]_{L=1}$$
$$= \frac{1}{w_k(1)}\left[w_k(L)\left(\frac{\beta_k'(L)}{\beta_k(L)} - \frac{\alpha'(L)}{\alpha(L)}\right)\right]_{L=1} = \frac{\beta_k'(1)}{\beta_k(1)} - \frac{\alpha'(1)}{\alpha(1)}, \tag{3}$$

where $z' = \frac{\partial z}{\partial L}$. Note, the mean lag associated with variable x_k does not depend on the coefficients of the other variables x_l, $l \neq k$.

In Eq. (3), $w_k(1)$ represents the long run impact of x_k on y. Consequently, $y_t - c^* - \sum_{k=1}^{n} w_k(1)x_{k,t}$ captures a deviation from the long-run equilibrium between the dependent variable y and regressors $x_1 \ldots x_n$. Following the steps outlined in Section 2.1 of Banerjee et al. (1993), the $ADL(p, q, n)$ model in Eq. (1) can be transformed into an $EC(p-1, q-1; n)$ model

$$\Delta y_t = \kappa\left[y_{t-1} - c^* - \sum_{k=1}^{n}\omega_k x_{k,t-1}\right] + \sum_{k=1}^{n} b_{k,0}\Delta x_{k,t}$$
$$+ a(L)\Delta y_t + \sum_{k=1}^{n} b_k(L)\Delta x_{k,t} + \epsilon_t, \tag{4}$$

where $\kappa = -\alpha(1)$, $\omega_k = w_k(1)$, $b_{k,0} = \beta_{k,0}$, $a(L) = \sum_{j=1}^{p-1} a_j L^j$ with $a_j = -\sum_{i=j+1}^{p}\alpha_i$, $b_k(L) = \sum_{j=1}^{q-1} b_{k,j}L^j$ with $b_{k,j} = -\sum_{i=j+1}^{q}\beta_i$, and $p-1$ and $q-1$ stand for the maximum lag lengths of Δy and Δx, respectively. By convention, a term does not enter the summation if the lower limit exceeds the upper limit. The models described by Eqs. (1) and (4) are isomorphic.

Example 1. Transformation of the $ADL(3, 3; 1)$ model

$$y_t = c + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \beta_0 x_t + \beta_1 x_{t-1}$$
$$+ \beta_2 x_{t-2} + \beta_3 x_{t-3} + \epsilon_t, \tag{5}$$

yields the following $EC(2, 2; 1)$ model

$$\Delta y_t = -(1-\alpha_1-\alpha_2-\alpha_3)\left[y_{t-1} - \frac{c}{1-\alpha_1-\alpha_2-\alpha_3}\right.$$
$$\left. - \frac{\beta_0+\beta_1+\beta_2+\beta_3}{1-\alpha_1-\alpha_2-\alpha_3}x_{t-1}\right] + \beta_0\Delta x_t$$
$$- (\alpha_2+\alpha_3)\Delta y_{t-1} - \alpha_3\Delta y_{t-2} - (\beta_2+\beta_3)\Delta x_{t-1}$$
$$- \beta_3\Delta x_{t-2} + \epsilon_t, \tag{6}$$

which can be estimated in a simplified form

$$\Delta y_t = \kappa\left[y_{t-1} - c^* - \omega x_{t-1}\right] + b_0\Delta x_t$$
$$+ a_1\Delta y_{t-1} + a_2\Delta y_{t-2} + b_1\Delta x_{t-1} + b_2\Delta x_{t-2} + \epsilon_t. \tag{7}$$

The expression in brackets represents the equilibrium error. The coefficients estimated in (7) can be mapped back to the ones in (5) and (6) with $\beta_0 = b_0$, $\beta_1 = b_1 - b_0 - \kappa\omega$, $\beta_2 = b_2 - b_1$, $\beta_3 = -b_2$, $\alpha_1 = 1 + \kappa + a_1$, $\alpha_2 = a_2 - a_1$, $\alpha_3 = -a_2$. Hence, for the $EC(2, 2; 1)$ model, the mean lag defined in Eq. (3) takes the following form

$$\mu = \frac{\beta_1 + 2\beta_2 + 3\beta_3}{\beta_0+\beta_1+\beta_2+\beta_3} + \frac{\alpha_1 + 2\alpha_2 + 3\alpha_3}{1-\alpha_1-\alpha_2-\alpha_3}$$
$$= \frac{\kappa\omega + b_0 + b_1 + b_2}{\kappa\omega} - \frac{1+\kappa-a_1-a_2}{\kappa}$$
$$= \frac{\omega(a_1+a_2-1) + b_0 + b_1 + b_2}{\kappa\omega}. \tag{8}$$

If $\kappa \neq 0$, $\omega_k \neq 0$, then consistent estimation of the parameters $\theta = (\kappa, \omega, a_1, a_2, b_0, b_1, b_2)'$ in Eq. (7) allows us to obtain a consistent estimate of the mean lag, $\hat{\mu} = \mu(\hat{\theta})$, and its variance $\text{Var}(\hat{\mu}) = \frac{\partial\mu(\hat{\theta})}{\partial\theta'}\text{Var}(\hat{\theta})\frac{\partial\mu(\hat{\theta})}{\partial\theta}$, where $\text{Var}(\hat{\theta})$ is the covariance matrix of coefficients estimated in Eq. (7).

Before generalizing this result to an $EC(p-1, q-1; n)$ model

$$\Delta y_t = \kappa\left[y_{t-1} - c^* - \sum_{k=1}^{n}\omega_k x_{k,t-1}\right] + \sum_{i=1}^{p-1} a_i\Delta y_{t-i}$$
$$+ \sum_{k=1}^{n}\sum_{j=0}^{q-1} b_{k,j}\Delta x_{k,t-j} + \epsilon_t, \tag{9}$$

we make the following set of assumptions:

Assumption 1. The variables $y, x_1 \ldots x_n$ entering models (1) and (9) are either jointly stationary, or cointegrated with a stationary equilibrium error $y_t - c^* - \sum_{k=1}^{n}\omega_k x_{k,t}$.

Assumption 2. The error in Eqs. (1) and (9), ϵ_t, is independently and identically distributed and is independent of the variables $x_1 \ldots x_n$.

Assumption 3. The parameters in Eq. (9), $\theta = (\kappa, \omega_1 \ldots \omega_n, a_1 \ldots a_{p-1}, b_{1,0} \ldots b_{1,q-1} \ldots b_{n,0} \ldots b_{n,q-1})'$, are estimated consistently with an estimator that has an asymptotically normal distribution $\sqrt{T}(\hat{\theta} - \theta) \xrightarrow{d} N(0, \text{Var}(\theta))$.

Assumption 4. $\kappa \neq 0$, $\omega_k \neq 0$ for $k \in \{1 \ldots n\}$. The mean lag, $\mu_k(\theta)$, is a continuous function of θ and is continuously differentiable with respect to θ.

Proposition 1. *Under Assumptions 1–4 the mean lag estimator*

$$\hat{\mu}_k = \mu_k(\hat{\theta}) = \frac{\hat{\omega}_k\left(\sum_{i=1}^{p-1}\hat{a}_i - 1\right) + \sum_{j=0}^{q-1}\hat{b}_{k,j}}{\hat{\kappa}\hat{\omega}_k}, \tag{10}$$

is consistent and asymptotically normally distributed

$$\sqrt{T}(\mu_k(\hat{\theta}) - \mu_k(\theta)) \xrightarrow{d} N\left(0, \frac{\partial\mu_k(\theta)}{\partial\theta'}\text{Var}(\theta)\frac{\partial\mu_k(\theta)}{\partial\theta}\right). \tag{11}$$

Proposition 1 extends the results obtained in Example 1 to a general $EC(p-1, q-1; n)$ model. The details of the proof are provided in an Online Supplement on the first author's homepage.

Remark 1.1. If the variables $y, x_1 \ldots x_n$ are cointegrated, then the elements of the cointegrating vector, $\omega = (\omega_1 \ldots \omega_n)'$, are estimated super-consistently: $\sqrt{T}(\hat{\omega} - \omega) = o_p(1)$. As a result, the $\text{Var}(\theta)$ components associated with the cointegrating vector, ω, converge to zero and do not contribute to the asymptotic variance of the mean lag estimator in (11). The Supplement contains a more detailed exposition of this issue (see Appendix A).

Perhaps due to previous unavailability of the formula presented in Eq. (10), some researchers have ignored the lag structure of