Economics Letters 148 (2016) 23–26

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

# Money and growth through innovation cycles with leisure

Jing Wan[a], Jie Zhang[b,c,*]

[a] East Asia Institute, National University of Singapore, Singapore 117570, Singapore
[b] School of Economics and Business Administration, Chongqing University, Chongqing, 400030, China
[c] National University of Singapore, Singapore 117570, Singapore

## HIGHLIGHTS

- Study monetary policy in a cash-in-advance model of growth through cycles with leisure.
- Higher money growth for lower income taxes increases labor, output, and growth immediately.
- It increases investment and average growth in output and variety on the period-two-cycle path.
- It amplifies fluctuations in investment, innovation, and output on the period-two-cycle path.
- Once high enough, it induces convergence on balanced growth with continuing innovations.

## ARTICLE INFO

*Article history:*
Received 7 November 2015
Received in revised form
18 May 2016
Accepted 11 September 2016
Available online 13 September 2016

*JEL classification:*
E3
E5
E6
D9

*Keywords:*
Money
Labor
Innovation
Investment
Growth
Cycles

## ABSTRACT

We study monetary policy with growth through innovation cycles and leisure. If consumption is cash constrained, increasing money growth for lower income taxes increases labor, output, investment, innovation, and growth and amplifies fluctuations on a period-two-cycle path. It induces convergence to the balanced-growth path at sufficiently high money growth rates. If investment for innovation and intermediate production is also cash constrained, the effects of money on labor, investment, innovation, and growth become negative at sufficiently high money growth rates.



## 1. Introduction

We study monetary policy by extending the Matsuyama (1999, 2001) model of growth through innovation cycles to incorporate leisure and a cash-in-advance constraint. We show that increasing money growth for lower income taxes increases labor, output, investment, innovation, and growth and amplifies fluctuations on a period-two-cycle path when consumption is cash constrained. Also, it induces convergence to the balanced-growth path at sufficiently high money growth rates. When investment for innovation and intermediate production is cash constrained as well, the effects of money on labor, investment, innovation, and growth become negative at sufficiently high money growth rates.

The results amplify the "Tobin effect" (Tobin, 1965) through increasing labor and innovation and permit persistent fluctuations with different patterns at different money growth rates. The Tobin effect is also amplified through externalities in Ho et al. (2007), Bhattacharya et al. (2009), and Lai and Chin (2010) or through Schumpeterian R&D in Chu and Cozzi (2014) and Chu and Ji (forthcoming), without fluctuations. Moreover, a negative effect of inflation on labor, innovation, and growth (reversed Tobin effects) emerges in Chu and Lai (2013).

* Correspondence to: Department of Economics, National University of Singapore, Singapore 117570, Singapore.
*E-mail addresses:* eaiwanj@nus.edu.sg (J. Wan), ecszj@nus.edu.sg, zhangjiecqu@cqu.edu.cn (J. Zhang).

http://dx.doi.org/10.1016/j.econlet.2016.09.009

The rest of the paper proceeds as follows. Section 2 introduces the model. Section 3 derives the results. Section 4 extends the cash-in-advance constraint to investment for innovation and intermediate production. Section 5 concludes the paper. An online appendix includes all proofs (see Appendix A).

## 2. The model

The economy has a mass $\hat{L}$ of identical, infinitely-lived agents, a final-good sector, and an intermediate sector.

### 2.1. Production and innovation

Final-good production uses labor $L_t$ and a composite of intermediates $x_t(z)$:

$$Y_t = \hat{A}(L_t)^{1/\sigma}\left\{\int_0^{N_t}[x_t(z)]^{1-\frac{1}{\sigma}}dz\right\}, \quad \hat{A} > 0,\ \sigma > 1. \tag{1}$$

One unit of intermediate product costs one unit of capital with a rental rate $r_t$. Old products $x_t^c \equiv x_t(z)$ for $z \in [0, N_{t-1}]$ have a competitive price $p_t^c = r_t$. New intermediates $x_t^m \equiv x_t(z)$ for $z \in [N_{t-1}, N_t]$, once introduced at $F$ units of capital, have a monopoly price $p_t^m = r_t\,\sigma/(\sigma-1)$ maximizing innovation profit $p_t^m x_t^m - r_t\left(x_t^m + F\right)$. Feasibility for innovation and intermediate production is

$$K_{t-1} = N_{t-1}x_t^c + (N_t - N_{t-1})\,(x_t^m + F). \tag{2}$$

Firms maximize profit such that $w_tL_t = (1/\sigma)Y_t$, $r_tK_{t-1} = (1-1/\sigma)Y_t$, and

$$\frac{x_t^c}{x_t^m} = \left(\frac{p_t^c}{p_c^m}\right)^{-\sigma} = \left(\frac{\sigma-1}{\sigma}\right)^{-\sigma}. \tag{3}$$

Free entry and breakeven for innovation imply

$$x_t^m \le (\sigma-1)\,F,\ N_t \ge N_{t-1},$$
$$\left[x_t^m - (\sigma-1)\,F\right](N_t - N_{t-1}) = 0. \tag{4}$$

From (2)–(4), the levels of intermediates and innovation are determined by:

$$x_t^c = \left(\frac{\sigma-1}{\sigma}\right)^{-\sigma} x_t^m = \min\left\{\frac{K_{t-1}}{N_{t-1}},\ \theta\sigma F\right\}, \tag{5}$$

$$N_t = N_{t-1} + \max\left\{0,\ \frac{K_{t-1}}{\sigma F} - \theta N_{t-1}\right\}, \tag{6}$$

where $\theta = (1-1/\sigma)^{1-\sigma} > 1$. Thus, total output equals

$$Y_t = \begin{cases} Al_t^{1/\sigma}\,[\theta\sigma FN_{t-1}]^{1/\sigma}\,[K_{t-1}]^{1-1/\sigma} \\ \quad \text{(Solow)} \quad \text{if } \dfrac{K_{t-1}}{N_{t-1}} \le \theta\sigma F, \\ Al_t^{1/\sigma}K_{t-1} \\ \quad \text{(Romer)} \quad \text{if } \dfrac{K_{t-1}}{N_{t-1}} > \theta\sigma F, \end{cases} \tag{7}$$

where $A \equiv \hat{A}\left[\hat{L}/(\theta\sigma F)\right]^{\frac{1}{\sigma}}$ and $l_t = L_t/\hat{L}$.

### 2.2. The government

Let $m_t$, $c_t$, and $p_t$ be the nominal money balance per agent, consumption per agent, and the price level, respectively. The government taxes income at rate $\tau_{yt}$ and consumption at rate $\tau_{ct}$ and issues money to finance lump-sum transfers $T_t$ as a fixed fraction $\xi$ of output:

$$T_t = \xi Y_t = \tau_{yt}\left(L_tw_t + r_tK_{t-1}\right) + \hat{L}c_t\tau_{ct} + \mu_t\frac{\overline{M}_{t-1}}{p_t} \tag{8}$$

where $\overline{M}_{t-1}$ is aggregate (nominal) money supply and $\mu_t \equiv (\overline{M}_t - \overline{M}_{t-1})/\overline{M}_{t-1}$ measures its growth. In equilibrium, $\overline{M}_t = M_t = \hat{L}m_t$.

### 2.3. Households

Household preference is:

$$\sum_{t=0}^{\infty}\beta^t\left[\ln c_t + \delta\frac{(1-l_t)^{1-\gamma}-1}{1-\gamma}\right], \quad \beta \in (0,1),\ \delta > 0,\ \gamma > 0. \tag{9}$$

With money balance $m_{t-1}$ and real asset $a_{t-1}$, an agent faces a budget constraint

$$c_t(1+\tau_{ct}) = \frac{m_{t-1}}{p_t} + (w_tl_t + r_ta_{t-1})\left(1-\tau_{yt}\right) + \frac{T_t}{\hat{L}} - \frac{m_t}{p_t} - a_t, \tag{10}$$

solvency $\lim_{t\to\infty} a_t/\prod_{s=0}^{t} r_s \ge 0$, and a cash-in-advance constraint $c_t(1+\tau_{ct}) \le m_{t-1}/p_t$.

## 3. Results

Under $1+\mu_t > \beta$, the equilibrium solution is given below:

$$\delta\,(1-l_t)^{-\gamma}\,l_t = \frac{\beta(1-\tau_{yt})}{\sigma\,(1+\mu_t)\,(1+\tau_{ct})\,[1-\beta(1-1/\sigma)(1-\tau_{yt+1})]}, \tag{11}$$

$$c_t = \left[1-\beta\left(1-\frac{1}{\sigma}\right)\left(1-\tau_{yt+1}\right)\right]Y_t/\hat{L}, \tag{12}$$

$$K_t = \beta\left(1-\frac{1}{\sigma}\right)\left(1-\tau_{yt+1}\right)Y_t, \tag{13}$$

$$p_t = \frac{M_{t-1}}{(1+\tau_{ct})\left[1-\beta\left(1-\frac{1}{\sigma}\right)\left(1-\tau_{yt+1}\right)\right]Y_t}, \tag{14}$$

$$\frac{m_t}{p_t} = (1+\mu_t)(1+\tau_{ct})\left[1-\beta\left(1-\frac{1}{\sigma}\right)\left(1-\tau_{yt+1}\right)\right]Y_t/\hat{L}, \tag{15}$$

along with (5)–(7), $w_tL_t = (1/\sigma)Y_t$, and $r_tK_{t-1} = (1-1/\sigma)Y_t$. For time-invariant rates of taxes and money growth, the proportional allocations of time and income become time-invariant. A time-invariant income tax rate is determined from the solution above and (8):

$$\tau_y = \frac{\xi - \left[1-\beta\left(1-\frac{1}{\sigma}\right)\right][\tau_c + \mu(1+\tau_c)]}{1+\beta\left(1-\frac{1}{\sigma}\right)[\tau_c + \mu(1+\tau_c)]}, \tag{16}$$

which is increasing in government lump-sum transfers and decreasing in money growth and consumption taxes. The short-run effects of money are:

**Proposition 1.** *Given $(K_{t-1}, N_{t-1}, \xi, \tau_c)$, a permanent increase in money growth for lower income taxes increases labor, output, investment, the capital-variety ratio for next-period innovation, and output growth in both regimes at $t$.*