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Estimation and test for quantile nonlinear cointegrating regression[☆]

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HIGHLIGHTS

- A quantile nonlinear cointegration model is proposed.
- The parameter estimator follows a nonstandard distribution asymptotically.
- A fully modified estimator and a test for linearity are developed.
- Monte Carlo results show that the test has good finite sample performance.

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ABSTRACT

In order to investigate the nonlinear relationship among economic variables at each quantile level, this paper proposes a quantile nonlinear cointegration model in which the nonlinear relationship at each quantile level is approximated by a polynomial. The parameter estimator in the proposed model is shown to follow a nonstandard distribution asymptotically due to serial correlation and endogeneity. Therefore, this paper develops a fully modified estimator which follows a mixture normal distribution asymptotically. Moreover, a test statistic for the linearity and its asymptotic distribution are also derived. Monte Carlo results show that the proposed test has good finite sample performance.

1. Introduction

It is well known that financial time series are leptokurtic and heavy tailed. While traditional time series models and approaches are no longer suitable for heavy tailed time series, much effort has been devoted to the time series quantile regression.[1] For stationary time series, Koenker and Xiao (2006) put forward a linear quantile autoregressive model, Chen et al. (2009) proposed a copula-based nonlinear quantile autoregressive model, and Galvao et al. (2010) developed a threshold quantile autoregressive model. Moreover, So and Chung (2015) examined the statistical properties of a two-step conditional quantile estimator in nonlinear time series models. In addition, Xiao and Koenker (2009) investigated the quantile regression estimation for the generalized autoregressive conditional heteroscedasticity (GARCH) model. For nonstationary time series, Koenker and Xiao (2004) developed the unit root quantile autoregression inference, and then was extended by Galvao (2009) to allow stationary covariates and a linear time trend. Li and Park (forthcoming) proposed a quantile nonlinear unit root test. More importantly, Xiao (2009) developed quantile cointegration models which have been widely used in finance and economics. Cho et al. (2015) extended Xiao's (2009) study and analyzed short-run dynamics and long-run cointegrating relationships across a range of quantiles by proposing a quantile autoregressive distributed lag model. Though the quantile cointegration model enables us to examine the quantile dependent cointegrating relationship among economic variables, it still assumes that the cointegrating relationship at each quantile level is linear and no one extended it to the nonlinear cointegrating framework. As Granger and Teräsvirta (1993) pointed out, however, "it was well known

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1 For an excellent review on time series quantile regression, see Xiao (2012a).

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that relationships between major economic variables were nonlinear and that nonlinear models abound in economic theory.'' Moreover, Hong and Phillips (2010) argued that neglecting the possible nonlinearity among nonstationary variables could lead to more serious consequences. Therefore, this paper extends Xiao's (2009) study to the nonlinear cointegrating framework.

To the best of our knowledge, this paper is the first to study the estimation and test for the quantile nonlinear cointegration even there is a vast literature on nonlinear cointegration, see Park and Phillips (1999), and Saikkonen and Choi (2004). Similar to Hong and Phillips (2010), we approximate the nonlinear function in each quantile level by a polynomial. For this reason, this paper also extends the polynomial cointegrating regression of Hong and Phillips (2010) to the quantile regressive framework. Even though polynomials can be used to approximate more general nonlinear functions well, it is restrictive in some sense. However, by the polynomial approximation, the model is linear in parameters after transforming so that linear quantile regression could be used to obtain the estimation, and therefore, we do not have to resort to the nonlinear optimization. In addition, we assume that regressors are endogenous and residuals are serially correlated. Therefore, a fully modified estimator is show to follow a mixture normal distribution asymptotically. Moreover, a test statistic is developed to test linearity in the proposed quantile cointegrating regression and its asymptotic distribution is derived. Monte Carlo results shows that our proposed test has good finite sample performance.

The quantile nonlinear cointegration models could be widely used in economics and finance. For example, it could be used to examine the relationship between the spot and futures prices and the interaction among financial markets. For example, by using Xiao's (2009) quantile cointegration model, Lee and Zeng (2011) investigated the relationship between the spot and futures oil prices of West Texas Intermediate and Burdekin and Siklos (2012) analyzed the contagion between Chinese, US and Asia-Pacific equity markets. However, it is hard to believe that the relationship between the spot and futures prices at each quantile level is linear. Moreover, it is also unrealistic to assume that the contagion effect among financial markets is linear. Therefore, the newly proposed nonlinear cointegration model could be used to provide more convincing analysis for these issues.

2. Quantile nonlinear cointegration and asymptotic properties

Xiao (2009) first developed the quantile cointegration model. Following Xiao (2009), we consider the following varying coefficient cointegration model:

$$y_t = \alpha + \beta_t' x_t + \mu_t, \tag{1}$$

where β_t is a monotone function of the innovation process. Let $F_\mu(\cdot)$ be the cumulative distribution function (c.d.f.) of μ_t, then Model (1) can be written as

$$Q_{y_t}(\tau|x_t) = \alpha + \beta(\tau)' x_t + F_\mu^{-1}(\tau), \tag{2}$$

where, $x_t = (x_{1t}, x_{2t}, \ldots, x_{kt})'$ is a k-dimensional vector of integrated variables, $Q_{y_t}(\tau|x_t)$ is the τth conditional quantile of y_t, $\beta(\tau)$ is a vector of parameters and may vary over the quantile levels. Model (2) assumes that the relationship between $\{y_t\}$ and $\{x_t\}$ is linear at each quantile level. Thus, it cannot capture the nonlinear relationship among variables. Therefore, we extend Xiao's (2009) model to the nonlinear framework:

$$Q_{y_t}(\tau|x_t) = g(x_t, \gamma(\tau)) + F_\mu^{-1}(\tau), \tag{3}$$

where $g(\cdot,\cdot)$ is a known nonlinear function and $\gamma(\tau)$ is a vector of parameters that may vary over the quantile levels. If $g(\cdot,\cdot)$ takes the form of smooth transition function, Model (3) includes the quantile smooth transition cointegrating model as its special case. However, Model (3) is such a general framework that it is quite hard to obtain the asymptotic properties if it is not impossible. Moreover, it is also hard to construct a test for linearity under the framework of Model (3).

Following Hong and Phillips (2010), we approximate $g(\cdot,\cdot)$ by a polynomial and obtain a quantile polynomial cointegrating regressive model:

$$Q_{y_t}(\tau|x_t) = \alpha + \beta(\tau)' x_t + \sum_{j=2}^{p} \gamma_j(\tau)' x_t^j + F_\mu^{-1}(\tau), \tag{4}$$

where $x_t^j = (x_{1t}^j, x_{2t}^j, \ldots, x_{kt}^j)'$, $j = 2, 3, \ldots, p$. Let $z_t = (1, x_t', x_t^{2'} \ldots x_t^{p'})'$, $\theta(\tau) = (\alpha(\tau), \beta(\tau)', \gamma(\tau)')'$, $\gamma = (\gamma_2'(\tau), \gamma_3'(\tau), \ldots, \gamma_p'(\tau))'$, $\gamma_j(\tau)$ is a k-dimensional vector of parameters that may vary over the quantile levels, $\alpha(\tau) = (\alpha + F_\mu^{-1}(\tau))$, then the estimator $\hat{\theta}(\tau)$ can be obtained by the following minimization problem:

$$\hat{\theta}(\tau) = \underset{\theta}{argmin} \sum_{t=1}^{T} \rho_\tau(y_t - \theta'(\tau) z_t). \tag{5}$$

Note that all parameters in Model (4) are quantile dependent. To obtain feasible estimators, the polynomial order p should be chosen in advance. On one hand, bigger p can approximate the $g(\cdot,\cdot)$ well. On the other hand, bigger p will increase the parameters estimated and reduce the degrees of freedom, and might cause spurious nonlinearity (Hong and Phillips, 2010). While some information criterions, such as AIC and BIC, could be employed to choose the polynomial order p, Hong and Phillips (2010) suggested $p = 2$ or 3 for which the Monte Carlo simulation gives better finite sample performance.

To develop the asymptotic theory, we impose the following assumptions.

Assumption 1. Let $v_t = (v_{1t}, v_{2t} \ldots v_{kt})' = \Delta x_t$ and μ_t be generated by

$$v_t = C_v(L)\varepsilon = \sum_{i=0}^{\infty} c_{vi}\varepsilon_{t-i}, \qquad \mu_t = C_u(L)\eta = \sum_{j=0}^{\infty} c_{uj}\eta_{t-j},$$

where

$$\sum_{i=0}^{\infty} i|c_{vi}| < \infty, \qquad \sum_{j=0}^{\infty} j|c_{uj}| < \infty, \qquad det((C_v(1))) \neq 0.$$

The vector process $\xi_t = (\varepsilon_t', \eta_t)'$ is a stationary and ergodic martingale difference sequence with natural filtration $\mathcal{F}_t = \sigma(\{\xi_s\}_{-\infty}^{t})$ satisfying that

(1) $E(\xi_t\xi_t'|\mathcal{F}_{t-1}) > 0$,

(2) $sup_{t\geq 1} E(\|\xi_t\|^r|\mathcal{F}_{t-1}) < \infty$, $a.s.$ for some $r > 4$.

Assumption 2. The disturbance μ_t is identically distributed with common c.d.f. $F(\mu)$, and has a continuous density $f(\mu)$ with $f(\mu) > 0$ on $\{\mu : 0 < f(\mu) < 1\}$.

Assumption 3. The conditional distribution function $F_{t-1}(\mu) = P(\mu_t < u|\mathcal{F}_{t-1})$ has derivative $f_{t-1}(\cdot), a.s.$, and $f_{t-1}(s_n)$ is uniformly integrable for any sequence $s_n \to F^{-1}(\tau)$. For some $\delta > 1$, $E[f_{t-1}^{\delta}(F^{-1}(\tau))] < \infty$.

Assumption 1 is standard in nonlinear cointegration time series analysis and is needed for the establishment of the multivariate invariance principle. Similar assumption can also be found in Hong and Phillips (2010). Assumptions 2 and 3 are also quite standard in