Economics Letters 148 (2016) 41–44

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Corporate bond pricing model with stochastically volatile firm value process

Woon Wook Jang [a,*], Young Ho Eom [b], Yong Joo Kang [b]

[a] *College of Government and Business, Yonsei University, 1 Yonseidae-gil, Wonju, Gangwon-Do, 220-710, South Korea*
[b] *School of Business, Yonsei University, 50 Yonsei-ro, Seodaemun-gu, Seoul 120-749, South Korea*

HIGHLIGHTS

- We propose a new structural model for corporate bond pricing.
- The model assumes the Heston (1993) stochastically volatile firm value process.
- It also assumes the Black and Cox (1976) before-maturity default possibility.
- The models potential is demonstrated using a simulation study.
- A semi-analytic solution method for the corporate bond prices is also provided.

ARTICLE INFO

Article history:
Received 21 July 2016
Received in revised form 13 September 2016
Accepted 17 September 2016
Available online 22 September 2016

JEL classification:
G10
G12
G13

Keywords:
Structural corporate bond pricing
Stochastic volatility
Fortet equation

ABSTRACT

We propose a new structural model for corporate bond pricing that assumes stochastically volatile firm value process with before-maturity default possibility. We demonstrate the model's potential using a simulation study and provide a semi-analytic solution method for the bond prices.

1. Introduction

In the seminal Merton (1974) model of structural corporate bond pricing, (1) firm value follows a geometric Brownian motion (GBM) and (2) default could occur (only) at maturity if firm value is less than the face value of the bond (hereafter, the "Merton default"). The empirical studies of the Merton model, however, report that it cannot generate sufficiently high yield spreads observed in the market data (see Eom et al., 2004).

Several models in structural approach have been proposed to overcome the shortcomings of the Merton model. This paper suggests a new corporate bond pricing model assuming that (1) the firm value process follows the stochastic volatility (SV) model of Heston (1993) with (2) the before-maturity default possibility assumption of Black and Cox (1976) (hereafter, the "Black–Cox default"), and present a semi-analytic solution method for the corporate bond prices.

The SV model has been successfully incorporated in the stock option pricing. However, due to a lack of tractability, particularly regarding default probability calculations (i.e. the first-passage-time density of the SV process) for coupon bonds, the credit risk model with SV process (with or without before-maturity defaults assumption) has rarely been touched. Meanwhile, interest in a SV firm value model has increased in the literature (Huang and Zhou, 2008; Zhang et al., 2009). Zhang et al. (2009), using a calibrated SV firm value model with the Merton default assumption, demonstrated the potential of the SV firm value model. This paper differs from Zhang et al. (2009) as we allow for default before the maturity date of the bond (Black–Cox default), which better reflects the actual situation of the credit markets.

* Corresponding author.
E-mail addresses: zara2k@yonsei.ac.kr (W.W. Jang), yheom@yonsei.ac.kr (Y.H. Eom), yongjoo.kang@yonsei.ac.kr (Y.J. Kang).

http://dx.doi.org/10.1016/j.econlet.2016.09.018
0165-1765/

In this paper we first conduct a simulation study to show the potential of our model. Then, we provide a semi-analytic solution method for the corporate bond prices.

2. Corporate bond pricing model with SV firm value process and before-maturity default

Let $A(t)$ be firm value at time t. Under the physical measure ($\mathbb{P}$) and the risk-neutral measure ($\mathbb{Q}$), we assume that firm value follows the SV process of Heston (1993) as follows for $i = \mathbb{P}$ or $\mathbb{Q}$;

$$\frac{dA(t)}{A(t)} = \left(\mu^i - \delta\right) dt + \sqrt{v(t)} dW_1^i(t) \text{ with } A(0) = A_0 \quad (1)$$

$$dv(t) = \kappa^i \left[\theta^i - v(t)\right] dt + \sigma\sqrt{v(t)} dW_2^i(t) \text{ with } v(0) = v_0 \quad (2)$$

$$\text{where } dW_1^i(t) dW_2^i(t) = \rho dt \quad (3)$$

where $\mu^{\mathbb{P}}$ is the instantaneous asset return, $\mu^{\mathbb{Q}} = r$ is the risk-free interest rate, and δ is the asset payout ratio. The asset return variance, $v(t)$, follows a square-root process with long-run mean θ^i, mean reversion speed κ^i , and volatility of variance σ. Finally, W_j^i, $j = 1, 2$, are two standard Brownian motions under the $\mathbb{P}$- and $\mathbb{Q}$-measure with correlation ρ. Here we define the asset risk premium π_a as $\pi_a = \mu^{\mathbb{P}} - r$ and the variance risk premium, ξ_v, is defined such that $\kappa^{\mathbb{Q}} = \kappa^{\mathbb{P}} + \xi_v$ and $\theta^{\mathbb{Q}} = \theta^{\mathbb{P}}\kappa^{\mathbb{P}}/\kappa^{\mathbb{Q}}$ as in Heston (1993).

For tractability we define the centered log-return of firm value as follows:

$$X(t) = \ln\left(\frac{A(t)}{A_0}\right) - \left(\mu^i - \delta\right) t, \text{ for } i = \mathbb{P} \text{ or } \mathbb{Q}.$$

Then, by Ito's lemma, Eq. (1) can be rewritten in terms of the variable $X(t)$ as follows:

$$dX(t) = -\frac{1}{2} v(t) dt + \sqrt{v(t)} dW_1^i(t) \text{ for } i = \mathbb{P} \text{ or } \mathbb{Q} \text{ with } X(0) = 0 \quad (4)$$

where the variance process $v(t)$ obeys the stochastic differential equation (2). We assume the Black–Cox default barrier K, so that default occurs when asset value $A(t)$ hits the barrier K, that is when $X(t)$ has approached the value $X^*(t) = \ln K/A_0 - \left(\mu^i - \delta\right) t$ at time $t \leq T$, where T is the maturity date of the debt.

3. Simulation results

To demonstrate the potential of the structural model with SV firm value process under the Black–Cox default assumption, we conducted a calibration analysis using Monte Carlo simulation.

We assume that corporate pure-discount bonds pay a proportion $(1 - w)$ of the face value of the bond at the maturity date if default occurs prior to the maturity. We define $Q(T; x_0, v_0) = \mathbb{E}_0^{\mathbb{Q}}\left[1_{\{\tau<T\}}\right]$ as the time-0 probability under the $\mathbb{Q}$-measure that default occurs before the bond maturity given that $X(0) = x_0$ and $v(0) = v_0$, where τ is the first-passage-time of firm value reaching the default boundary.

The time-0 pure-discount bond price with maturity date T, $D(T; x_0, v_0)$, assuming the face value of the bond is 1, is then given by:

$$D(T; x_0, v_0) = e^{-rT}\mathbb{E}_0^{\mathbb{Q}}\left[1 - w1_{\{\tau<T\}}\right] = e^{-rT}\left[1 - wQ(T; x_0, v_0)\right].$$

We price coupon bonds as a portfolio of pure-discount bonds, but with different write-down rate for coupon, w_c.

We apply the model across rating categories of high investment grade (A), low investment grade (BBB), and speculative grade (BB). The model parameter values are obtained from Zhang et al. (2009) and presented in Table 1.

Table 1
Base model parameter values (Zhang et al., 2009).

Rating category	Rating A	Rating BBB	Rating BB
Asset volatility $\sqrt{v_0}$ (%)	21.65	25.69	25.70
Debt face value K	43.13	48.02	58.63
Asset risk premium π_a (%)	3.49	3.97	2.92
Variance risk premium ξ_v (%)	−1.59	−1.53	−1.44
Mean reversion $\kappa^{\mathbb{P}}$	0.74	0.72	0.42
Long-run mean of variance $\theta^{\mathbb{P}}$ (%)	4.24	4.75	4.90
Vol. of variance σ (%)	4.01	4.53	5.36
Correlation ρ (%)	−24.02	−28.42	−27.13
	Common variable values		
Risk-free rate, r (%)		5	
Asset payout ratio, δ (%)		2	
Coupon rate (%)		7.5	
Write-down rate of face value, w (%)		56	
Write-down rate of coupon, w_c (%)		100	

Fig. 1 compares the term structure of credit spreads when firm value follows SV process with different default assumptions (the Merton versus Black–Cox default assumption). In each figure we depict credit spreads for both the pure-discount bonds and coupon bonds.

For all rating categories, the model using the Black–Cox default assumption creates larger credit spreads, roughly double with our parameter values, than that obtained using the Merton assumption. Specifically, under the Merton default assumption credit spreads for 5-year maturity coupon bonds for rating A, BBB, and BB are respectively 39, 95, and 214 basis points. However, when we apply the Black–Cox default assumption we obtain 75, 189, and 435 basis points respectively for each of the rating categories, which are closer to the credit spreads observed in the market. In fact, Zhang et al. (2009) showed that credit spreads for the 5-year maturity bonds are about 60, 140, and 430 basis points for ratings A, BBB, and BB, respectively.

Next, we conducted sensitivity analyses to observe the effects of each variance process parameter on the credit spreads. We increase and decrease the base parameter values in Table 1 by 5% and name it high- and low-parameter values, respectively. Results are shown in Fig. 2.

Fig. 2 suggests that credit spreads are most sensitive to changes in the initial volatility level, v_0, and long run mean of volatility, $\theta^{\mathbb{P}}$, parameters while the effects of the other parameters are rather moderate. More interestingly, the three credit spread curves in the upper right most panel suggests that a humped shaped credit curve can be obtained for the speculative grade bond (BB rating) when the initial volatility level, v_0, is high while an upward sloping credit curve is obtained when it is at a low level.

4. A semi-analytic solution method for the corporate bond prices

We suggest the calculation of $Q(T; x_0, v_0)$ by applying Fortet's (1943) equation for two-dimensional Markov processes. Proposition 2 of Collin-Dufresne and Goldstein (2001) (hereafter, CDG) derives Fortet's equation for two-dimensional Markov processes and proposed corporate bond pricing method under the mean reverting leverage ratio process model, which are joint Gaussian. However, two factors within our model are non-Gaussian with no closed-form solution for the transition probability density.

We denote $p(x_t, v_t, t|x_s, v_s, s)$ as the joint transition probability density of x and v at time t given the state of x_s and v_s at time s. The following proposition gives the pricing method as a discretization of the two-dimensional Fortet's equation if $p(x_t, v_t, t|x_s, v_s, s)$ is given, and proof is omitted since it is almost the same as that for Proposition 2 of CDG. The only difference is that we need to replace the Gaussian densities in equations (38) and (39) of CDG with the