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Stock market participation and endogenous boom-bust dynamics

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HIGHLIGHTS

- We study the impact of stock market participation on the dynamics of stock markets.
- Stock market participation depends on the market's price trend and on its mispricing.
- Increasing stock market participation may produce endogenous boom-bust dynamics.
- The predictions of our model are in line with empirical and experimental evidence.

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ABSTRACT

We develop a model in which stock market participation depends on current market movements and on the fundamental state of the market. Our model explains empirical and experimental evidence according to which increasing (decreasing) stock market participation amplifies bubbles (crashes).

1. Introduction

Stock markets regularly display severe bubbles and crashes, which, in turn, may have dramatic consequences for the real economy. Prominent examples include the stock market crash of 1929, leading to the Great Depression, and the stock market crash of 2007, leading to the Great Recession. Empirical evidence indicates that changes in stock market participation amplify boom-bust dynamics. Kindleberger and Aliber (2005, p. 12) conclude that during a stock market hike, there is a pervasive sense among investors that "it is time to get on the train before it leaves the station". Shiller (2015, p. 70), likewise reports that "investors, their confidence and expectations buoyed by past price increases, bid up speculative prices further, thereby enticing more investors to do the same, so that the cycle repeats again and again".

Similar conclusions can be drawn from laboratory experiments. Kirchler et al. (2015) observe that the joint inflow of cash and investors leads to massive overvaluations before prices crash towards the asset's maturity, even though investors have complete information about the asset's (constant) fundamental value. However, Noussair and Tucker (2016) show that bubbles and crashes are also common in settings in which the initial quantity of cash is sufficiently high.

Inspired by these observations, we develop a simple evolutionary market entry model in which a market maker adjusts the stock price with respect to the current stock demand. The market entry decision of investors depends on current market movements and the fundamental state of the market, i.e. investors tend to enter (exit) the market in periods of increasing (decreasing) stock prices and exit (enter) the market in periods of overvaluation (undervaluation). As it turns out, the dynamics of stock prices is due to a two-dimensional nonlinear map. The model has an inner steady state in which the stock price corresponds to the discounted value of future dividends. We analytically show that the inner steady state becomes unstable (i) if investors' market entry decisions rely strongly on past price changes, (ii) if the pool of potential investors is large or (iii) if investors' stock demand is high. Since numerical investigations furthermore reveal that endogenous boom-bust dynamics are set in motion if the inner steady state becomes unstable,

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our model explains the aforementioned empirical and experimental observations.

Our model is part of a literature stream, surveyed in Hommes (2013), which describes the dynamics of financial markets by the behavior of heterogeneous, boundedly rational and interacting traders. The amplification mechanism outlined in our paper may be added to such models. Our paper also contributes to the literature stream which replicates laboratory experiments with agent-based models. Anufriev and Hommes (2012) have presented a powerful example in this direction. In the rest of our paper, we introduce the model (Section 2), present our analytical (Section 3) and numerical results (Section 4), and offer some conclusions (Section 5).

2. An evolutionary market entry model

The stock market is populated by a market maker and a time-varying number of investors. The market maker sets the stock price P_t with respect to investors' current stock demand

$$P_t = aD_t, \tag{1}$$

where a is a positive parameter and D_t stands for investors' stock demand. Taking the difference between $P_t = aD_t$ and $P_{t-1} = aD_{t-1}$ and solving this relation for P_t reveals that $P_t = P_{t-1} + a(D_t - D_{t-1})$, i.e. (1) is consistent with the classical market maker framework (Chiarella et al., 2009). Accordingly, stock prices increase if investors buy additional stocks ($D_t > D_{t-1}$) while stock prices decrease if they sell part of their stocks ($D_t < D_{t-1}$).

Similar as in Lux and Marchesi (1999), the stock demand of an active investor is constant and given by $b > 0$. Let n_t be the number of active investors. Investors' total stock demand can then be expressed by

$$D_t = bn_t. \tag{2}$$

Setting $\alpha = ab$, (1) and (2) yield

$$P_t = abn_t = \alpha n_t, \tag{3}$$

i.e. stock prices depend positively on stock market participation. Shiller (2015) argues that a typical investor's decision whether to enter the stock market is not always based on careful calculations. Instead, investors show a tendency to enter the market when stock prices increase. This may be because they become increasingly optimistic or simply to prevent feelings of regret about missing out on speculative profits.[1] However, Shiller (2015) also argues that investors know that stock markets cannot grow forever. There are dampening factors – quantitative anchors such as dividend-price ratios – which eventually become relevant. We summarize this view by defining the stock market's attractiveness as

$$A_t = \beta\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right) + \gamma\left(\frac{\overline{D}}{P_t} - r\right). \tag{4}$$

The first term on the right-hand side of (4) indicates the stock market's speculative gain potential, where β is a positive parameter. The stronger the current stock price increases (decreases), the more (less) attractive the stock market appears. The second term captures the stock market's fundamental gain potential relative to an investment in a safe asset, where γ is a positive parameter, $\overline{D}$ denotes the stock market's constant average dividend payments, and r reflects the return of a safe asset. Increasing (decreasing) stock prices decrease (increase) the relative fundamental gain potential of the stock market, making it less (more) attractive.

We use exponential replicator dynamics (Hofbauer and Sigmund, 1988) to describe the number of active investors

$$n_t = N\frac{n_{t-1}}{n_{t-1} + (N - n_{t-1})Exp[-\lambda A_{t-1}]}, \tag{5}$$

where N stands for the total number of investors and λ represents the investors' intensity of choice. Note that an increase in the relative attractiveness of the stock market leads to an increase in stock market participation and that the increase in stock market participation is stronger as the investors' intensity of choice increases.[2]

3. Analytical results

By introducing the auxiliary variable $x_t = n_{t-1}$ and combining (3)–(5), we can express our model by the two-dimensional nonlinear map

$$T: \begin{cases} n_{t+1} = N\dfrac{n_t}{n_t + (N - n_t)Exp\left[-\lambda\left(\beta\left(\frac{n_t - x_t}{x_t}\right) + \gamma\left(\frac{\overline{D}}{\alpha n_t} - r\right)\right)\right]} \\ x_{t+1} = n_t. \end{cases} \tag{6}$$

Since we set the scaling parameter λ to 1, the dynamics depends solely on $\alpha, \beta, \gamma, N, \overline{D}$ and r.

Straightforward computations reveal that (6) may give rise to three steady states. With respect to the steady states of the active number of investors we have $\overline{n}_1 = 0 < \overline{n}_2 = \frac{\overline{D}}{\alpha r} < \overline{n}_3 = N$, implying that $\overline{P}_1 = 0 < \overline{P}_2 = \frac{\overline{D}}{r} < \overline{P}_3 = \alpha N$. The two border steady states are economically uninteresting which is why we focus on the inner steady state. $\overline{P}_2 = \frac{\overline{D}}{r}$ implies that the inner steady state price is given by the discounted value of future dividends. This requires that exactly $\overline{n}_2 = \frac{\overline{D}}{\alpha r}$ investors must enter the market. Additional (fewer) investors make the stock market less (more) profitable than the safe asset.

To determine the stability of the inner steady state, we derive the characteristic polynomial from the Jacobian matrix of (6), i.e.

$$J(\overline{n}_2, \overline{x}_2) = \begin{pmatrix} \dfrac{Nr\alpha(1+\beta - r\gamma) + \overline{D}(r\gamma - \beta)}{Nr\alpha} & \left(\dfrac{\overline{D}}{Nr\alpha} - 1\right)\beta \\ 1 & 0 \end{pmatrix} \tag{7}$$

and obtain

$$z^2 - z\frac{Nr\alpha(1+\beta - r\gamma) + \overline{D}(r\gamma - \beta)}{Nr\alpha} - \left(\frac{\overline{D}}{Nr\alpha} - 1\right)\beta = 0. \tag{8}$$

It can be shown that the eigenvalues of the characteristic polynomial are less than one in modulus if

$$\gamma < \gamma_c = \frac{2N}{r(N - \overline{n}_2)} + \frac{2\beta}{r} \tag{9}$$

and

$$\beta < \beta_c = \frac{N}{N - \overline{n}_2} \tag{10}$$

simultaneously apply. According to (9), the inner steady state becomes unstable if γ crosses γ_c, a situation which leads to a flip

[1] Kindleberger and Aliber (2005, p. 29), write that "There is nothing as disturbing to one's well-being and judgment as to see a friend get rich. Unless it is to see a nonfriend get rich".

[2] More precisely, n_t denotes the expected number of active investors and thus n_t is not an integer variable. However, it is more convenient (and common) to use the term 'active number of investors'.