Economics Letters 145 (2016) 6–10

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A symmetric two-player all-pay contest with correlated information[☆]

Zhiyang Liu[a], Bo Chen[b,c,*]

[a] School of International Business Administration, Shanghai University of Finance and Economics, China
[b] Department of Economics, Southern Methodist University, USA
[c] School of Economics, Zhejiang University, China

HIGHLIGHTS

- We study a two-player all-pay auction with binary types and correlated information structures.
- We characterize both monotonic and non-monotonic symmetric Bayesian Nash equilibria for the all-pay auction game.
- The symmetric equilibrium is monotonic if the types are mildly positively/negatively correlated and is otherwise non-monotonic.
- We employ parametric distributions to illustrate the symmetric equilibrium.

ARTICLE INFO

Article history:
Received 29 February 2016
Received in revised form
25 April 2016
Accepted 6 May 2016
Available online 19 May 2016

JEL classification:
C72
D44
D72
D82

Keywords:
All-pay auction
Correlated types
Symmetric equilibrium

ABSTRACT

We construct both monotonic and non-monotonic symmetric Bayesian Nash equilibria for a two-player all-pay contest with binary types and correlated information structures. We also employ a class of parametric distributions to illustrate our equilibrium construction explicitly and to derive some comparative statics results.

1. Introduction

Contests, where players invest irreversible resources to win a reward, are abundant and essential in social and economic life. Contest models hence have found various applications in social sciences. Sports competitions, patent races, promotion tournaments in organizations, and election campaigns among politicians are just a few examples that can be fruitfully modeled and analyzed as contests.[1]

We study a perfectly discriminatory contest (all-pay auction) where two ex ante identical players exert effort to compete for a prize. Each player has private information about her effort efficiency, which affects her effort cost and can be either high or low. The players' effort efficiencies are drawn from a symmetric joint distribution with full support. Importantly, the players' effort efficiencies can be statistically (negatively or positively) correlated. To put the model into context, consider a promotion competition between two colleagues. The colleagues have private information about their own abilities. However, due to past interactions and collaborations, such private information may not be statistically independent.

In this contest game, we characterize the symmetric Bayesian Nash equilibrium for *all* symmetric joint distributions with binary and correlated types. We find that the symmetric equilibrium is monotonic when the players' types are mildly (positively or negatively) correlated, while the symmetric equilibrium is

☆ We are grateful to the editor and especially an anonymous referee for very constructive comments and advice. Remaining errors are ours.

* Corresponding author at: Department of Economics, Southern Methodist University, USA.
E-mail addresses: lzy3919@126.com (Z. Liu), bochen@smu.edu (B. Chen).

1 Konrad (2009) gives a comprehensive survey of the literature on contests.

http://dx.doi.org/10.1016/j.econlet.2016.05.004

non-monotonic when the types are sufficiently (positively or negatively) correlated.[2] We also use a convenient class of distributions with a parametrized correlation parameter to illustrate our equilibrium construction, the equilibrium strategies, and some comparative statics results.

Our paper is related to the literature on all-pay auctions with incomplete information. Several important papers have studied monotonic equilibria in various settings with discrete/continuous and independent/correlated types (e.g., Amann and Leininger, 1996, Krishna and Morgan, 1997, Konrad, 2004, and Siegel, 2014). While we consider a simple model with binary and correlated types, our focus is on both monotonic and non-monotonic equilibria. Our study is hence more closely related to two recent contributions Lu and Parreiras (2014), who analyze conditions for the existence of monotonic equilibria in a general setting with correlated (continuous) types and interdependent valuations *and* non-monotonic equilibria in a setting with quadratic valuations, and Rentschler and Turocy (Forthcoming) who present a useful algorithm for symmetric monotonic *and* non-monotonic equilibria in a setting with discrete signals and interdependent valuations. While our setting is admittedly simpler and more special in comparison, we completely construct symmetric monotonic and non-monotonic equilibria for *all correlation structures* in our setting, and are hence able to delineate the relationship between correlation and the equilibrium type more explicitly.

2. Model

Consider an all-pay contest where two risk-neutral players exert effort to compete for a single prize of value $V > 0$. Prior to the competition, each player $i \in \{1, 2\}$ observes a private signal about her ability (or effort efficiency) $\theta_i \in \{H, L\}$ which is referred to as i's type (with slight abuse of notation) and $H > L > 0$. It is commonly known that the joint probability distribution of the types is $\Pr(\theta_i, \theta_{-i})$ which is symmetric and has full support (conditional probabilities $\Pr(\theta_{-i}|\theta_i)$ are hence well defined and positive):

$$\Pr(H, L) = \Pr(L, H) \quad \text{and}$$
$$\Pr(\theta_i, \theta_{-i}) > 0 \quad \text{for all } (\theta_i, \theta_{-i}) \in \{H, L\}^2 . \qquad (1)$$

Given (1), we have for example $\Pr(\theta_1 = H|\theta_2 = L) = \Pr(\theta_2 = H|\theta_1 = L)$. We hence use the shorthand notation "$\Pr(H|H)$, $\Pr(L|L)$, $\Pr(H|L)$, $\Pr(L|H)$" hereafter.

To illustrate the distribution in (1), consider the class of distributions in Fig. 1.

	H	L
H	$\alpha^2 + \rho\alpha(1-\alpha)$	$(1-\rho)\alpha(1-\alpha)$
L	$(1-\rho)\alpha(1-\alpha)$	$(1-\alpha)^2 + \rho\alpha(1-\alpha)$

Fig. 1. An example for the distribution $\Pr(\theta_i, \theta_{-i})$ in (1).

Here $\alpha \in (0, 1)$ is the ex ante probability of type H, while $\rho \in \left(\max\left\{-\frac{\alpha}{1-\alpha}, -\frac{1-\alpha}{\alpha}\right\}, 1\right)$ is the linear correlation between the types.[3] The types' interim beliefs are:

$$\Pr(H|H) = \alpha + \rho(1-\alpha) \quad \text{and}$$
$$\Pr(L|H) = 1 - \Pr(H|H), \qquad (2)$$
$$\Pr(H|L) = \alpha(1-\rho) \quad \text{and} \quad \Pr(L|L) = 1 - \Pr(H|L).$$

After observing their signals, the players simultaneously choose non-negative efforts. The cost of effort $e_i \in [0, +\infty)$ for player i with ability θ_i is $c_i(e) = \frac{e_i}{\theta_i}$ and player i's payoff from effort profile (e_i, e_{-i}) is hence

$$u_i(e_i, e_{-i}|\theta_i) = \begin{cases} V - \dfrac{e_i}{\theta_i} & \text{if } e_i > e_{-i}, \\ pV - \dfrac{e_i}{\theta_i}, \quad p \in [0,1] & \text{if } e_i = e_{-i}, \\ -\dfrac{e_i}{\theta_i} & \text{if } e_i < e_{-i}. \end{cases}$$

The above all-pay contest is equivalent to an all-pay auction with two ex ante symmetric bidders, private valuations, and correlated types where e_i is player i's bid.

3. Analysis

The all-pay contest with the information structure in (1) admits a symmetric Bayesian Nash equilibrium where each type of a player randomizes over a (connected) interval of efforts. We denote the equilibrium as $\sigma^* = (\sigma_1^*(\theta), \sigma_2^*(\theta))$ where $\theta \in \{H, L\}$, $\sigma_1^*(\theta) = \sigma_2^*(\theta)$, and $\sigma_i^*(\theta) = F_\theta(\cdot)$ is player i's behavioral strategy which is a probability distribution over Borel measurable subsets of $E = [0, HV]$.

Proposition 1. *Consider the contest with the correlated information structure in (1). There is a symmetric Bayesian Nash equilibrium where both types play a mixed strategy:*

(i) *If* $\frac{H}{L} \geq \frac{\Pr(L|L)}{\Pr(L|H)}$ *and* $\frac{H}{L} \geq \frac{\Pr(H|L)}{\Pr(H|H)}$, *the symmetric equilibrium is* ***monotonic***:

$$F_L(e) = \frac{e}{\underline{e}} \quad \text{for } e \in [0, \underline{e}] \quad \text{and}$$
$$F_H(e) = \frac{e - \underline{e}}{\Pr(H|H)\,HV} \quad \text{for } e \in [\underline{e}, \bar{e}],$$
$$\underline{e} = \Pr(L|L)\,LV \quad \text{and} \quad \bar{e} = \underline{e} + \Pr(H|H)\,HV.$$

(ii) *If* $\frac{\Pr(L|L)}{\Pr(L|H)} > \frac{H}{L}$, *the symmetric equilibrium is* ***non-monotonic***:

$$F_H(e) = \begin{cases} \dfrac{(\Pr(L|L)\,L - \Pr(L|H)\,H)\,e}{(\Pr(L|L) - \Pr(L|H))\,HLV} & \text{for } e \in [0, \underline{e}] \\ \dfrac{e - \Pr(L|H)\,HV}{\Pr(H|H)\,HV} & \text{for } e \in [\underline{e}, \bar{e}], \end{cases}$$

$$F_L(e) = \frac{(\Pr(H|H)\,H - \Pr(H|L)\,L)\,e}{(\Pr(L|L) - \Pr(L|H))\,HLV} \quad \text{for } e \in [0, \underline{e}],$$

$$\underline{e} = LV\frac{(\Pr(L|L) - \Pr(L|H))\,H}{\Pr(H|H)\,H - \Pr(H|L)\,L} \quad \text{and} \quad \bar{e} = HV.$$

(iii) *If* $\frac{H}{L} < \frac{\Pr(H|L)}{\Pr(H|H)}$, *the symmetric equilibrium is* ***non-monotonic***:

$$F_H(e) = \frac{(\Pr(L|H)\,H - \Pr(L|L)\,L)\,e - \Pr(L|L)\,(H-L)\,\bar{e}}{(\Pr(L|H) - \Pr(L|L))\,H\bar{e}},$$
$$\text{for } e \in [\underline{e}, \bar{e}],$$

$$F_L(e) = \begin{cases} \dfrac{e}{\Pr(L|L)\,\bar{e}}, & \text{for } e \in [0, \underline{e}] \\ \dfrac{(\Pr(H|L)\,L - \Pr(H|H)\,H)\,e + \Pr(H|L)\,(H-L)\,\bar{e}}{(\Pr(H|L) - \Pr(H|H))\,H\bar{e}}, \\ \text{for } e \in [\underline{e}, \bar{e}], \end{cases}$$

$$\underline{e} = \frac{\Pr(L|L)\,(H-L)\,\bar{e}}{\Pr(L|H)\,H - \Pr(L|L)\,L} \quad \text{and} \quad \bar{e} = LV.$$

[2] Our symmetric equilibrium is always in mixed strategies. Roughly, the equilibrium is monotonic if different types of a contestant randomize on non-overlapping intervals, while the equilibrium is non-monotonic if different types of a contestant randomize on overlapping intervals.

[3] The types can be negatively correlated. However, for the probabilities in Fig. 1 to be well-defined, we impose the restriction that $\rho > \max\left\{-\frac{\alpha}{1-\alpha}, -\frac{1-\alpha}{\alpha}\right\}$.