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Indeterminacy and history dependence of strategically interacting players

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HIGHLIGHTS

- Extension of Krugman's (1991) labor market for strategic instead of competitive players.
- Krugman's results carry over to strategic players if sufficiently many participate.
- The cooperative solution exhibits a few non-standard features.
- Indeterminate longrun and transient outcomes are possible even for few players.

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ABSTRACT

This paper considers a finite number of agents populating Krugman's (1991) labor market. The objective is to investigate whether the much emphasized indeterminate outcome is due to the assumption of uncountable many agents, each of measure 0. It is shown that this result extends to n players each with strategic leverage if the social reference includes the own action. This multiplicity results in an open loop setting, which renders in almost all other cases a unique intertemporal Nash equilibrium. Finally, the cooperative solution exhibits non-standard features: the possibility of converging to the (stationary inferior) agricultural equilibrium and that is due to an unstable node while an unstable spiral can render the unique outcome of full industrialization.

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1. Introduction

This paper replaces competitive by strategically interacting agents in Krugman's (1991) labor market: each worker has to decide whether to expand activity in manufacturing, subject to increasing returns to scale, or in agriculture, subject to constant returns. It is shown that open loop strategies of n strategic players lead history dependent and to potentially indeterminate outcomes. This stresses that atomistic behavior is not a prerequisite for indeterminacy, suggesting that indeterminacy or multiplicity is an intrinsic feature of the history versus expectations model rather than the nature of information underpinning the solution concept. This is surprising as the open loop setting renders in most cases a unique intertemporal Nash equilibrium in contrast with the almost generic multiplicity of equilibria in Markov strategies (Tsutsui and Mino, 1990, Dockner and van Long, 1993, Rowat, 2007 and Wirl, 2007).

2. Model

Identical agents, $i = 1, \ldots, n$, solve the intertemporal optimization problem,

$$\max_{u_i} \int_0^\infty \exp(-rt)\left[\pi\left(x_i(t), X(t)\right) - \frac{c}{2}u_i^2(t)\right]dt,\ \ i = 1, \ldots, n, \tag{1}$$

$$\dot{x}_i(t) = u_i(t),\ \ x_i(0) = x_{i0}. \tag{2}$$

Payoff π depends on the private state (x_i) and the social aggregate X. Control (u_i) is costly and for simplicity quadratic. Krugman (1991) considers the payoff (assuming symmetry allows to drop

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the index i),

$$\pi(x, X) = x w_m(X) + (1 - x) w_a, \tag{3}$$

where w_a and w_m are the wages paid in agriculture (constant) and in manufacturing,

$$w_m(X) = \alpha + \beta X, \quad 0 < \alpha < w_a, \ \beta > 0, \alpha + \beta > w_a; \tag{4}$$

x = time an individual spends in manufacturing. The aggregate X is given by the average,

$$X(t) = \frac{1}{n}\sum_{i=1}^{n} x_i(t), \tag{5}$$

and

$$0 < \tilde{X} := (w_a - \alpha)/\beta < 1 \tag{6}$$

is the level at which manufacturing pays the same wage as agriculture, $w_m\left(\tilde{X}\right) = w_a$. (4) stipulates that manufacturing pays less in an agricultural (X small) but more in an industrialized economy (X large) due to increasing returns to scale in the latter.[1] Individual labor supply is fixed and normalized to 1, thus

$$x_i \in [0, 1] \, \forall i. \tag{7}$$

Although the state space is the product of $n+1$ unit intervals, one for each player plus one for X, restriction to symmetric equilibria reduces the state space to (x, X) with the equilibrium determined at the 45° line, $x = X$. What makes this differential game (1)–(7) special is the bounded state space. Therefore, the usual linear strategies are not potential equilibrium candidates in spite of a linear–quadratic differential game.

3. Competitive Krugman (1991)

Krugman (1991) assumes – infinitely many agents with an aggregate of 1 and symmetry, thus $X = x$ – and derives: The two differential equations for the labor share in manufacturing and the shadow price of time allocated to manufacturing (λ),

$$\dot{x} = \frac{\lambda}{c}, \tag{8}$$

$$\dot{\lambda} = r\lambda - \alpha - \beta x + w_a, \tag{9}$$

characterize the competitive rational expectation equilibrium. The unique interior steady state,

$$\tilde{X} = \frac{w_a - \alpha}{\beta}, \tag{10}$$

is unstable. Therefore, the longrun outcome is at the boundary, either $x \to 0$ or $x \to 1$. Which one depends on the initial condition x_0 and in addition on expectations if the eigenvalues of the Jacobian,

$$\varepsilon_{12} = \frac{1}{2}\left(r \pm \sqrt{r^2 - \frac{4\beta}{c}}\right), \tag{11}$$

are complex.

An unstable spiral (or a focus) results, iff $4\beta > r^2 c$. This leads to an overlap of the policies and thus requires information in addition to the initial condition x_0 to determine the intertemporal competitive equilibrium transiently and in the longrun, i.e., whether $x \to 0$ or $x \to 1$. Krugman (1991) links expectation driven outcomes to complex eigenvalues as an if and only if characterization. Actually, the domain of indeterminacy can include unstable nodes, an example is in Caulkins et al. (2014).

4. Cooperation

The first best maximizes the payoff of the individual (representative) agent accounting for the spillover due to an agent's investments, i.e., substituting $x = X$. Applying the Hamilton–Jacobi–Bellman equation leads to a functional equation for the value function (W),

$$rW = \max_u \left\{x(\alpha + \beta x) + (1 - x) w_a - \frac{c}{2}u^2 + W'u\right\}. \tag{12}$$

Substituting the value maximizing the right hand side of (12),

$$u = \frac{W'}{c}, \tag{13}$$

yields

$$rW = x(\alpha + \beta x) + (1 - x) w_a + \frac{1}{2}\frac{W'^2}{c}. \tag{14}$$

The value function allows to determine which path is optimal if multiple solutions satisfy the first order conditions from applying optimal control theory.

Defining the (current value) Hamiltonian,

$$H = \pi(x, x) - \frac{c}{2}u^2 + \mu u = x(\alpha + \beta x) + (1 - x) w_a + \mu u, \tag{15}$$

the first order conditions are,

$$\dot{x} = u = \frac{\mu}{c}, \tag{16}$$

$$\dot{\mu} = r\mu - \alpha - 2\beta x + w_a. \tag{17}$$

The canonical equations ($\dot{x}$, $\dot{\mu}$) have a unique and unstable steady state (identified by hat and superscript c for cooperative),

$$0 < \hat{x}^c = \frac{w_a - \alpha}{2\beta} < \tilde{X}, \quad \hat{\mu} = 0, \tag{18}$$

which is below the competitive counterpart (10). The eigenvalues of the Jacobian of the canonical equations system are

$$\epsilon_{12} = \frac{1}{2}\left(r \pm \sqrt{r^2 - \frac{8\beta}{c}}\right). \tag{19}$$

Iff $8\beta > r^2 c$ an unstable spiral results and thus for a less steep slope (β) and/or higher costs (c) as in the competitive case. However, optimization rules out generic indeterminacy, yet the optimal choice, $x \to 0$ or 1, can depend on the initial conditions.

Proposition 1. *Cooperation lowers the unstable steady state below the competitive one and is an unstable spiral whenever perfect competition induces an unstable spiral. In spite of an interior steady state, even if it is an unstable spiral, global convergence to the manufacturing economy can be optimal. However, $\alpha + \beta > w_a$, does not guarantee that manufacturing is the first best longrun outcome.*

A transition to manufacturing is optimal for the example in Fig. 1 in which adjustment costs and discount rate are relatively

[1] Krugman (1991) claims that the two characteristics – increasing returns and externalities – are crucial for multiple longrun equilibria and complexities in this simple labor market model; Wirl and Feichtinger (2006), however, show that positive and strong social interactions ($\pi_{xX} = w'_m > 0$ and $|\pi_{xX}/\pi_{xx}| > 1$, which are even infinite in (3)) are the crucial conditions and that non-increasing returns to scale are neither necessary nor sufficient.

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