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Is it Brownian or fractional Brownian motion?

Meiyu Li [a], Ramazan Gençay [a,*], Yi Xue [b]

[a] Department of Economics, Simon Fraser University, 8888 University Drive, Burnaby, BC, V5A 1S6, Canada

[b] Department of Finance, School of International Trade and Economics, University of International Business and Economics, 10 Huixin Dongjie, Chaoyang District, 10029, Beijing, China

HIGHLIGHTS

- We propose test statistics for testing Brownian motion against fractional Brownian motion.
- Our test framework is robust to finite large jumps.
- We extend bi-power variation to the inference of Hurst index.

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ABSTRACT

Fractional Brownian motion embeds Brownian motion as a special case and offers more flexible diffusion component for pricing models. We propose test statistics based on bi-power variation for testing Brownian motion against fractional Brownian motion alternatives. To filter out the prevalent existence of finite large jumps, a truncation method based on Hurst index estimator is proposed. Simulation results confirm the consistency of jump truncation framework with desirable empirical size and viable empirical power for our tests.

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1. Introduction

Empirical investigations have provided mass evidence against the traditional asset pricing model based on Brownian motion (Bm), dB_r, where returns are independent normal. Possible long range dependence discovered in financial market returns stimulates the application of fractional Brownian motion (fBm). On one hand, modeling fBm at return level directly, dB_r^H where H stands for Hurst index, provides an alternative explanation for some stylized puzzles presented by classical models. For instance, Rostek and Schöbel (2006) delivers an analytical form of implied volatility with assumptions of constant instantaneous volatility and dB_r^H to explain the volatility smile and smirk. The shape of the implied volatility over time to maturity can be taken as a result of correlated returns. Moreover, due to the latency associated with the different speeds of information diffusion captured by time varying Hurst index H_t, Rostek (2014) concludes that temporary under reaction or over-reaction might be an alternative reason for volatility clustering. On the other hand, neglecting the possible dependent returns and pretending Bm as the underlying data generating process may have serious drawbacks. For instance, the presence of long range dependence forces CAPM to unrealistically assume identical horizon for all investors (Greene and Fielitz, 1980).

With such concerns, we propose tests of Bm against fBm based on bi-power variation. In addition, we also delineate a two-step jump truncation method to filter out the existence of finite large jumps in financial returns. Our simulation results confirm the consistency of jump truncation framework with desirable empirical size and viable empirical power for our proposed tests.

* Corresponding author.
E-mail addresses: meiyul@sfu.ca (M. Li), rgencay@sfu.ca (R. Gençay), yxue@uibe.edu.cn (Y. Xue).

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2. Jump adjustment of financial data

2.1. Model setting

In both Sections 2 and 3, we follow the notations of Barndorff-Nielson and Shephard (2006) to discuss the data preprocessing of finite large jumps and our central limit theorems for Bm test.

Let $Y(t)$ denotes the logarithmic price of a financial asset, which follows fractional Brownian motion jump diffusion process $\{FBMJ\}$:

$$Y(t) = \int_0^t \mu(s)\mathrm{d}s + \int_0^t \sigma(s)\mathrm{d}B^H(s) + \sum_{j=1}^{N(t)} \kappa(s_j)$$

where the mean process $\mu(t)$ is continuous and of finite variation, $\sigma(t) > 0$ denotes the càdlàg instantaneous volatility. Instead of Bm ($H = 0.5$), we assume fBm with the Hurst index $H \in (0, 1)$. The $N(t)$ process counts the number of jumps occurring with possibly time-varying intensity $\lambda(t)$ and jump size $\kappa(s_j)$. This process contains correlated continuous movement generated by fBm and large infrequent random discontinuous jumps. It captures the property of long range dependence, skewness, leptokurtic and fat tails, with major information arriving infrequently and randomly. Through this paper, we assume the following two conditions.

Condition 1. *The volatility process σ is càdlàg and path-wise bounded away from* 0.

Condition 2. *The joint process (μ, σ) is independent of the fBm B^H.*

We construct δ-returns as the difference of discrete Y with intervals of time length $\delta > 0 y_j = Y_{j\delta} - Y_{(j-1)\delta}$, $j = 1, 2, \ldots, \lfloor t/\delta \rfloor$. The *realized quadratic variation* process is defined as $[Y_\delta]_t = \sum_{j=1}^{\lfloor t/\delta \rfloor} y_j^2$, while the *realized* $\{1, 1\}$*-order bi-power variation* process is $\{Y_\delta\}_t^{[1,1]} = \sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}|\,|y_j|$.

2.2. Jump elimination by truncation

To filter out the impact of large jumps, we need following two steps:

1. Find the jump robust estimation of Hurst index value, $\hat{H}(Jump)$.
2. Keep only the increments y_j if $|y_j| < \alpha\delta^{\varpi}$ for some constant $\varpi \in (0, \hat{H}(Jump))$ and $\alpha > 0$.

Barndorff-Nielson and Shephard (2004) showed that if $Y \in FBMJ$ with $H = 0.5$, and Conditions 1 and 2 hold, then $\sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}|\,|y_j| \xrightarrow{p} \mu_1^2 \int_0^t \sigma_s^2 \mathrm{d}s$, where $\mu_1 = E|u| = \sqrt{2}/\sqrt{\pi}$ and $u \sim N(0, 1)$. Duan and Xue (2014) extended this result to fBm, and showed that if $Y \in FBMJ$ with $\forall H \in (0, 1)$, and Conditions 1 and 2 hold, then $\sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}|\,|y_j| \xrightarrow{p} \mu_1^2 f(\rho_1) \int_0^t \sigma_s^2 \mathrm{d}s$ and $\sum_{j=3}^{\lfloor t/\delta \rfloor} |y_{j-2}|\,|y_j| \xrightarrow{p} \mu_1^2 f(\rho_2) \int_0^t \sigma_s^2 \mathrm{d}s$, where $f(\rho) = (1 - \rho^2)^{0.5} + \rho * Arctan\left(\rho/(1-\rho^2)^{0.5}\right)$ and $\rho_1(H) = 2^{2H-1} - 1$, $\rho_2(H) = (3^{2H} + 1)/(2^{2H+1}) - 1$ are functions of H.[1]

Duan and Xue (2014) proposes the following jump robust ratio estimation for Hurst index

$$R(\hat{H}(Jump)) \equiv \frac{f(\rho_1(\hat{H}(Jump)))}{f(\rho_2(\hat{H}(Jump)))} = \frac{\sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}|\,|y_j|}{\sum_{j=3}^{\lfloor t/\delta \rfloor} |y_{j-2}|\,|y_j|} \xrightarrow{p} R(H) \equiv \frac{f(\rho_1(H))}{f(\rho_2(H))}.$$

[1] The correlation coefficient function of $\mathrm{d}B_t^H$ and $\mathrm{d}B_{t+n}^H$ is $\rho_n(H) = \frac{1/2[(n+1)^{2H} + (n-1)^{2H} - 2n^{2H}]}{n^{2H}}$.

Once $\hat{H}(Jump)$ is obtained, we define the threshold for data truncation as $\alpha\delta^{\varpi}$ for some constant $\varpi \in (0, \hat{H}(Jump))$ and $\alpha > 0$. By setting $\varpi < \hat{H}(Jump)$, we asymptotically keep only the increments that mainly consist of fBm components by eliminating finite large jumps. Moreover, the restriction on the rate at which threshold approaches 0 depends on the choice of ϖ (Aït-Sahalia and Jacod, 2012).[2] In summary, the two steps proposed in this section asymptotically eliminate the impact of large jumps and leaving behind the continuous process.[3]

3. Hurst index test based on bi-power variation

The jump truncation framework allows us to asymptotically focus on the price process, *FBM*, with only continuous fBm diffusion, $Y(t) = \int_0^t \mu(s)\mathrm{d}s + \int_0^t \sigma(s)\mathrm{d}B^H(s)$.

3.1. Theorem with $H = 0.5$

Without large finite jumps, we find that the jump test theorem in Barndorff-Nielson and Shephard (2006) can be used to test Bm against fBm, as to whether $H = 0.5$.

Theorem 3.1 (*Barndorff-Nielson and Shephard, 2006*)**.** *Let $Y \in FBM$ with $H = 0.5$ and let t be a fixed, arbitrary time. Suppose Conditions 1 and 2 hold:*

Then as $\delta \downarrow 0$

$$GM = \frac{\delta^{-1/2}\left(\mu_1^{-2}\sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}|\,|y_j| - \sum_{j=1}^{\lfloor t/\delta \rfloor} y_j^2\right)}{\sqrt{\vartheta_1 \int_0^t \sigma_s^4 \mathrm{d}s}} \xrightarrow{L} N(0, 1)$$

where $\vartheta_1 = \frac{\pi^2}{4} + \pi - 5$, $\mu_1 = E|u| = \sqrt{2}/\sqrt{\pi}$.

Under the alternative $H \neq 0.5$ (fBm), $f(\rho_1) \neq 1$ leads to a nonzero numerator, implying deviation from $N(0, 1)$ under the null hypothesis of $H = 0.5$ (Bm).

Similarly, due to the fact that $f(\rho_1) = f(\rho_2) = 1$ under null hypothesis of $H = 0.5$ (Bm), we propose another theorem to test Bm against fBm based on bi-power variation as follows

Theorem 3.2. *Let $Y \in FBM$ with $H = 0.5$ and let t be a fixed, arbitrary time. Suppose Conditions 1 and 2 hold:*

Then as $\delta \downarrow 0$

$$BM = \frac{\delta^{-1/2}\mu_1^{-2}\left(\sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}|\,|y_j| - \sum_{j=3}^{\lfloor t/\delta \rfloor} |y_{j-2}|\,|y_j|\right)}{\sqrt{\vartheta_2 \int_0^t \sigma_s^4 \mathrm{d}s}} \xrightarrow{L} N(0, 1)$$

where $\vartheta_2 = \frac{\pi^2}{2} - 2\pi + 2$, $\mu_1 = E|u| = \sqrt{2}/\sqrt{\pi}$.

Again, under the alternative $H \neq 0.5$ (fBm), $f(\rho_1) \neq f(\rho_2)$ leads to a nonzero numerator, implying deviation from $N(0, 1)$ under the null hypothesis of $H = 0.5$ (Bm).

[2] Aït-Sahalia and Jacod (2012) provides a summary for data truncation based on Bm. Here, we extend this method to a more general setting with fBm.

[3] Lee and Hannig (2010) proposes a test to detect both large and small jumps in Lévy jump diffusion processes, which is based on the assumption of standard Brownian motion. However, we need to remove the jumps under a more general framework, fractional Brownian motion, to deliver the efficiency of BM_J and validity of BH_J. Therefore, we have to restrict our model to the one with only finite large jumps instead of general Lévy jumps to fit our jump truncation method.

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