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Precautionary self-insurance-cum-protection[☆]

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HIGHLIGHTS

- Precautionary self-insurance-cum-protection is examined.
- Additive/multiplicative background risk is introduced.
- Monotone comparative statics and risk apportionment are used.
- Prudence is required for precautionary self-insurance-cum-protection.

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ABSTRACT

Precautionary self-insurance-cum-protection (SICP) arises when an individual spends more on SICP when background risk is introduced. We develop a two-period model wherein additive/multiplicative background risk prevails in the second period. Using the theory of monotone comparative statics and risk apportionment, we derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. Prudence is called for to create a precautionary motive that induces the individual to shift his wealth in a way to reduce the loss of expected utility caused by the addition of background risk, thereby giving rise to the precautionary SICP.

1. Introduction

The seminal work of Ehrlich and Becker (1972) examines how individuals facing insurable risk of a loss can invest in activities that either reduce the size of the loss (self-insurance) or the probability of the loss (self-protection). Lee (1998) points out that many actions taken by individuals for risk management purposes may provide both self-insurance and self-protection at the same time, which he refers to as self-insurance-cum-protection (SICP).[1] Examples of SICP include the use of high quality brakes that reduces both the probability of an automobile accident and the resulting damages, the practice of regular medical checkups that decreases the probability and severity of an illness, and many others.

Precautionary SICP arises when individuals spend more on SICP upon the addition of background risk. In this paper, we develop a two-period model wherein an individual invests in SICP in the first period to manage insurable risk of a loss that occurs in the second period.[2] We introduce background risk in the second period, where this risk is independent of the insurable risk, and can be either additive (e.g., random wealth) or multiplicative (e.g., inflation risk) in nature. Using the theory of monotone comparative statics (Milgrom and Shannon, 1994) and risk apportionment (Eeckhoudt and Schlesinger, 2006; Eeckhoudt et al., 2009a,b), we derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. We show that prudence is

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[1] Indeed, Ehrlich and Becker (1972) recognize SICP and give an example wherein good lawyers are able to reduce not only the probability of conviction but also the punishment for crime.

[2] Our two-period model can also be interpreted as a single-period model as in Lee (2012) by treating the first-period utility as the utility cost arising from SICP.

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required for the precautionary SICP. There is a precautionary motive that induces the prudent individual to shift his wealth from the first period to the second period when the background risk is introduced. Doing so reduces the "pain" caused by the background risk in the second period, where "pain" is defined as the loss of expected utility (Eeckhoudt and Schlesinger, 2006).

Our results generalize those of Eeckhoudt et al. (2012) and Lee (2012) to the case of precautionary SICP under additive background risk, without relying on the first- and second-order conditions for the individual's decision problem. We further derive novel results of precautionary SICP under multiplicative background risk, which contributes to the understanding of multiplicative risk apportionment (Wang and Li, 2010).

The rest of this paper is organized as follows. Section 2 delineates our two-period model of SICP in the presence of additive/multiplicative background risk. Sections 3 and 4 derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk deteriorates via higher-order stochastic dominance. The final section concludes.

2. The model

Consider a two-period model of an individual who has initial wealth w_0 in the first period and w_1 in the second period. The individual has utility functions, $v(w)$ and $u(w)$, defined over his first-period and second-period final wealth, respectively, where both $v(w)$ and $u(w)$ are continuously differentiable functions of w. Let T be a positive integer and define $U_T \equiv \{u(w) : (-1)^{n+1}u^{(n)}(w) > 0$ for $n = 1, \ldots, T\}$, where $u^{(n)}(w) = \mathrm{d}^n u(w)/\mathrm{d}w^n$ denotes the nth derivative of $u(w)$. Hence, U_T is the set of utility functions that exhibit mixed risk aversion up to order T (Caballé and Pomansky, 1996).

While there is no uncertainty in the first period, the individual faces insurable risk, $\tilde{z}$, and background risk, $\tilde{\varepsilon}$, in the second period. The background risk, $\tilde{\varepsilon}$, is either additive (e.g., random wealth) or multiplicative (e.g., inflation risk) in nature. The two random variables, $\tilde{z}$ and $\tilde{\varepsilon}$, are independent of each other.

In contrast to the background risk, $\tilde{\varepsilon}$, which is neither hedgeable nor insurable, the insurable risk, $\tilde{z}$, can be managed by the individual by spending an amount, e, on self-insurance-cum-protection (SICP). The expenditure, e, on SICP is endogenously chosen from the compact set, $[0, w_0]$, by the individual in the first period. There are two loss events in the second period, "loss" and "no loss", which partition the state space into the "loss states" and the "no-loss states", respectively. Given that e has been spent on SICP, $\tilde{z} = \ell(e)$ in the loss states, which occurs with probability $p(e)$, and $\tilde{z} = 0$ in the no-loss states, which occurs with probability $1 - p(e)$, where $0 < \ell(e) < w_1$ and $0 < p(e) < 1$ for all $e \in [0, w_0]$. We assume that the individual's SICP is effective in that more expenditure on SICP reduces both the magnitude of loss and the probability of the loss event, i.e., both $\ell(e)$ and $p(e)$ are decreasing functions of e.

3. Additive background risk

In this section, we examine the case that the background risk, $\tilde{\varepsilon}$, is additive in nature such that $\tilde{\varepsilon}$ is a zero-mean random variable. The individual's expected utility over the two periods is given by

$$f(e) = v(w_0 - e) + p(e)\mathrm{E}\{u[w_1 - \ell(e) + \tilde{\varepsilon}]\} + [1 - p(e)]\mathrm{E}[u(w_1 + \tilde{\varepsilon})] \tag{1}$$

where $\mathrm{E}(\cdot)$ is the expectation operator. The individual's ex-ante decision problem is to choose $e \in [0, w_0]$ so as to maximize $f(e)$. Since $f(e)$ is a continuous function of e, the set, $\arg\max_{e\in[0,w_0]} f(e)$, is non-empty, and plausibly not a singleton. Let e^* be an element in $\arg\max_{e\in[0,w_0]} f(e)$.

The individual is said to demonstrate precautionary SICP if he spends more on SICP in the presence than in the absence of the zero-mean additive background risk. To derive conditions under which precautionary SICP prevails, we examine the case that the background risk, $\tilde{\varepsilon}$, changes to $\tilde{\xi}$, where $\tilde{\xi}$ is a random variable that is dominated by $\tilde{\varepsilon}$ via the Nth-order stochastic dominance and $N \geq 1$. In this case, the individual's two-period expected utility becomes

$$g(e) = v(w_0 - e) + p(e)\mathrm{E}\{u[w_1 - \ell(e) + \tilde{\xi}]\} + [1 - p(e)]\mathrm{E}[u(w_1 + \tilde{\xi})]. \tag{2}$$

The individual's ex-ante decision problem is to choose $e \in [0, w_0]$ so as to maximize $g(e)$. Since $g(e)$ is a continuous function of e, the set, $\arg\max_{e\in[0,w_0]} g(e)$, is non-empty, and plausibly not a singleton. Let e^{**} be an element in $\arg\max_{e\in[0,w_0]} g(e)$.

To derive necessary and sufficient conditions under which the individual spends more on SICP when the background risk becomes more risky, i.e., $e^{**} \geq e^*$, we have to compare the two sets, $\arg\max_{e\in[0,w_0]} f(e)$ and $\arg\max_{e\in[0,w_0]} g(e)$. This falls into a principal concern in the theory of monotone comparative statics (Milgrom and Shannon, 1994). We state and prove the following lemma.

Lemma 1. *Consider two random variables, $\tilde{\varepsilon}$ and $\tilde{\xi}$, such that $\tilde{\varepsilon}$ dominates $\tilde{\xi}$ via the Nth-order stochastic dominance. The following condition holds:*

$$\mathrm{E}[u(w + k + \tilde{\xi})] - \mathrm{E}[u(w - k + \tilde{\xi})] > \mathrm{E}[u(w + k + \tilde{\varepsilon})] - \mathrm{E}[u(w - k + \tilde{\varepsilon})], \tag{3}$$

for all scalars, $k > 0$, if, and only if, the utility function, $u(w)$, exhibits mixed risk aversion up to order $N + 1$, i.e., $u(w) \in U_{N+1}$.

Proof. According to Theorem 3 of Eeckhoudt et al. (2009b), the 50–50 binary lottery, $[\tilde{\varepsilon} - k; \tilde{\xi} + k]$, dominates the 50–50 binary lottery, $[\tilde{\varepsilon} + k; \tilde{\xi} - k]$, in the sense of $(N + 1)$th-order stochastic dominance. Hence, we have

$$\frac{1}{2}\mathrm{E}[u(w - k + \tilde{\varepsilon})] + \frac{1}{2}\mathrm{E}[u(w + k + \tilde{\xi})] > \frac{1}{2}\mathrm{E}[u(w + k + \tilde{\varepsilon})] + \frac{1}{2}\mathrm{E}[u(w - k + \tilde{\xi})], \tag{4}$$

if, and only if, $u(w) \in U_{N+1}$. Rearranging terms of inequality (4) yields condition (3). □

Lemma 1 shows preferences for harm disaggregation (Eeckhoudt and Schlesinger, 2006) in that individuals with $u(w) \in U_{N+1}$ prefer the 50–50 binary lottery, $[\tilde{\varepsilon} - k; \tilde{\xi} + k]$, to the 50–50 binary lottery, $[\tilde{\varepsilon} + k; \tilde{\xi} - k]$. The two harms, replacing $\tilde{\varepsilon}$ by $\tilde{\xi}$ and k by $-k$, are better apportioned in the former lottery than in the latter lottery in the sense that they never jointly appear in each of the two states of nature.

Using Lemma 1 and the theory of monotone comparative statics, we derive necessary and sufficient conditions under which $e^{**} \geq e^*$ in the following proposition.

Proposition 1. *Given that the zero-mean additive background risk, $\tilde{\varepsilon}$, experiences an increase in risk to $\tilde{\xi}$ in the sense of Nth-order stochastic dominance, the individual spends more on SICP, i.e., $e^{**} \geq e^*$, if, and only if, the individual's utility function, $u(w)$, exhibits mixed risk aversion up to order $N + 1$, i.e., $u(w) \in U_{N+1}$.*

Proof. For any $e_1 > e_0$, it follows from Eq. (1) that $f(e_1) - f(e_0) \geq 0$ is equivalent to

$$p(e_1)\mathrm{E}\{u[w_1 - \ell(e_1) + \tilde{\varepsilon}]\} - p(e_0)\mathrm{E}\{u[w_1 - \ell(e_0) + \tilde{\varepsilon}]\} + [p(e_0) - p(e_1)]\mathrm{E}[u(w_1 + \tilde{\varepsilon})] \geq v(w_0 - e_0) - v(w_0 - e_1). \tag{5}$$