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Models of limited self-control: Comparison and implications for bargaining☆

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HIGHLIGHTS

- Fudenberg and Levine's dual-self model (2006) is compared with β-δ discounting.
- Fudenberg and Levine (FL) agents care about future self-control costs.
- β-δ agents can be viewed as FL agents that do not care about such costs.
- The models' differing implications are compared in a bargaining game.

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ABSTRACT

This paper compares two models of limited intertemporal self-control: the linear-cost version of Fudenberg and Levine's dual-self model (2006) and the quasi-hyperbolic discounting model. The main distinction between the two frameworks can be formulated as whether agents care about future self-control costs: dual selves do, while quasi-hyperbolic discounters do not. The dual-self model is applied to a bargaining game with alternating proposals where players negotiate over an infinite stream of payoffs, and it is shown that, in subgame-perfect equilibrium, the first proposer's payoff is unique and agreement is immediate. By contrast, Lu (2016) shows that with quasi-hyperbolic discounters, a multiplicity of payoffs and delay can arise in equilibrium.

1. Introduction

To explain preference reversals that are likely caused by inconsistent preferences over time,[1] economists and psychologists have put forth the idea that immediate rewards are disproportionately more appealing than rewards in the near, but not immediate future. *Quasi-hyperbolic* discounting, where the sequence of discount factors is $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots$ with $\beta, \delta \in (0, 1)$, is often used to capture this extra weight put on immediate payoffs.[2]

An alternative framework for studying limited self-control is Fudenberg and Levine's (2006) dual-self model.[3] It postulates that each agent is comprised of a sequence of short-run selves interacting with the world and a long-run self that may, at a cost, influence the short-run self.[4] Each short-run self lasts only one

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1 Frederick et al. (2002) provide an overview of some experimental findings.

2 Phelps and Pollak (1968) first proposed this discount function to study intergenerational saving, and Laibson (1997) applied it to individual intertemporal decision-making. See, for example, Angeletos et al. (2001) and Laibson et al. (2007) for empirical support, and Gul and Pesendorfer (2005) and Montiel Olea and Strzalecki (2014) for axiomatic foundations.

3 Many other dual-self models have been proposed, *e.g.* Thaler and Shefrin (1981), Bénabou and Pycia (2002), Loewenstein and O'Donoghue (2004), Bernheim and Rangel (2004), Benhabib and Bisin (2005) and Brocas and Carrillo (2008). In this paper, Fudenberg and Levine's model is used due to its generality (notably, it applies to situations with an infinite horizon, unlike some of the models listed above) and its tractability. Also, Fudenberg and Levine show that, with linear self-control costs (as assumed in this paper), their model satisfies the axioms from Gul and Pesendorfer (2001).

McClure et al. (2004) show, through functional magnetic resonance imaging (fMRI), that there are two distinct brain systems governing discounting, which provides a motivation for dual-self models.

4 More precisely, whenever it is an agent's turn to move, the long-run self acts first by deciding whether and how much to change the short-run self's preferences; the latter self then moves in the main game.

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period and cares only about the immediate payoff, while the long-run self discounts the future with a standard exponential function.

This paper studies the relation between the dual-self model and quasi-hyperbolic discounting. Proposition 1 in Section 2 shows that sophisticated quasi-hyperbolic agents[5] can be understood as dual selves with self-control costs linear in the amount of immediate utility forgone, but modified such that the long-run self no longer cares about the costs of influencing future short-run selves, even though she is aware of them.[6] Therefore, the main distinction between the two frameworks is whether agents care about their future self-control costs.

Section 3 shows that this difference can have a large impact on equilibrium predictions in games. The example used is the alternating-offer bargaining game proposed by Lu (2016), where an *infinite stream* of unit-surpluses is divided, unlike in Ståhl (1972) and Rubinstein (1982). Each offer allocates the entire stream. The game ends when an offer is accepted; when an offer is rejected, that period's surplus is lost. The fact that both current and future surpluses are divided corresponds to many economic situations (*e.g.* employment), and is important for teasing out the effects of limited self-control: when proposing, agents are tempted to demand more of the current surplus (*e.g.* in the form of a signing bonus) in exchange for future surplus.[7] Proposition 2 describes subgame-perfect equilibrium (SPNE) play between dual selves with equal discount factor δ. Here, agreement is always immediate, and the first proposer's payoff is unique and continuous in the self-control parameters; Lu (2016) shows that neither is true with quasi-hyperbolic discounters.

2. Relation between the dual-self model and quasi-hyperbolic discounting

Fudenberg and Levine (2006) propose a *dual-self* model where: (i) each person acts through a sequence short-run selves that each cares only about utility in the current period, and (ii) a forward-looking long-run self, before the short-run self plays in each period, can take actions affecting how the short-run self's choice determines current utility. They show that under mild assumptions,[8] their dual-self model has an equivalent reduced form where the long-run self directly takes actions to maximize aggregate utility at time t given by

$$U_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t}(u_\tau - C_\tau),$$

where u_τ is the utility of the short-run self at time τ, and C_τ is the self-control cost incurred by the long-run self at time τ. This paper adopts its most tractable form, where the cost to the long-run self of making the short-run self take action a when the state variable is y, denoted $C_t(y, a)$, is linear in the difference in short-run utility, $u_t(y, .)$, caused by the change:

$$C_t(y, a) = \gamma \left[\sup_{a'} \; u_t(y, a') - u_t(y, a) \right],$$

where $\gamma > 0$.

To relate quasi-hyperbolic discounting and the dual-self model, define, as a technical device, the following modified version of the dual self:

Definition. A *selfish dual self* is a dual self whose long-run self's utility at time t is $U_t = -C_t + \sum_{\tau=t}^{\infty} \delta^{\tau-t} u_\tau$, where u_τ is the utility of the short-run self at time τ, and C_t is the self-control cost incurred by the long-run self at time t.

The difference between the regular dual self and the selfish dual self is that the latter does not care about self-control costs $C_{t+1}, C_{t+2}, \ldots$ incurred by future versions of himself, and therefore only cares about the presence of future temptation if it affects future choices. The preferences of the long-run self are therefore time-inconsistent themselves. Proposition 1 shows that the utility of the selfish dual self directly relates to that of the quasi-hyperbolic discounter.

Proposition 1. *Suppose an agent chooses from a set of streams* $\mathbf{u}^k$ *of expected utility, where* u_t^k *denotes the expected utility from stream* k *at time* t. *Let the valuation of* $\mathbf{u}^k$ *by a quasi-hyperbolic agent with discount function* $1, \beta\delta, \beta\delta^2, \ldots$ *be* u_{QH}^k, *and let the valuation of* $\mathbf{u}^k$ *by a selfish dual self with linear self-control cost coefficient* γ *be* u_{DS}^k. *Then, if* $\beta = \frac{1}{1+\gamma}$, $u_{DS}^k = -\gamma \sup_{k'}\{u_0^{k'}\} + (1+\gamma)u_{QH}^k$ *for all* k.

Proof. Denote the current period as period 0. We have $u_{QH}^k = u_0^k + \beta \sum_{t=1}^{\infty} \delta^t u_t^k = u_0^k + \frac{1}{1+\gamma} \sum_{t=1}^{\infty} \delta^t u_t^k$.

The dual self's self-control cost of choosing stream k is $C_0^k = \gamma(\sup_{k'}\{u_0^{k'}\} - u_0^k)$. It follows that

$$\begin{aligned} u_{DS}^k &= -\gamma \left(\sup_{k'}\{u_0^{k'}\} - u_0^k \right) + \sum_{t=0}^{\infty} \delta^t u_t^k \\ &= -\gamma \sup_{k'}\{u_0^{k'}\} + (1+\gamma)u_0^k + \sum_{t=1}^{\infty} \delta^t u_t^k \\ &= -\gamma \sup_{k'}\{u_0^{k'}\} + (1+\gamma)u_{QH}^k. \quad \square \end{aligned}$$

Since u_{DS}^k is an affine transformation of u_{QH}^k, Proposition 1 states that, under the parametrization $\beta = \frac{1}{1+\gamma}$, the quasi-hyperbolic discounter and the selfish dual self have the same preferences. By ignoring future self-control costs, the selfish long-run self, just like the quasi-hyperbolic agent, treats two future periods in a time-consistent way, but treats today and tomorrow differently than two future periods.[9] Example 1 in the Appendix illustrates the result.

3. Bargaining between dual selves

3.1. The game

Two players with transferable utility bargain in discrete time over an infinite stream of unit surpluses. The game starts in period 0, and in each even (odd) period t, player 1 (2) proposes an

5 Sophistication means that agents know (and are not mistaken about) their future selves' preferences.

6 This "long-run self" would therefore be more accurately described as a sequence of forward-looking agents. For brevity, however, the term "long-run self" is used. Xue (2008) shows that quasi-hyperbolic discounting can be obtained as the result of cooperative bargaining between a myopic self and a more patient time-consistent self.

7 It can be shown that with dual-self agents whose discount factors are δ_i and whose self-control costs are linear with coefficients γ_i, the SPNE is unique and the same as with exponential agents whose discount factors are $\delta_i/(1+\gamma_i)$. Kodritsch (2014) shows that, with quasi-hyperbolic agents, the same holds with "effective" exponential discount factors $\beta_i\delta_i$. Therefore, in SPNE, self-control problems, as defined in either the quasi-hyperbolic or the dual-self framework, cannot be separated from time-consistent impatience in complete-information Rubinstein–Ståhl bargaining.

8 Namely, self-control is costly, the long-run self is able to make the short-run self take any action, utility is continuous in both selves' actions, and the long-run self can break ties faced by the short-run self at arbitrarily small cost.

9 I thank an anonymous referee for suggesting this remark.