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Interpreting heterogeneous coefficient spatial autoregressive panel models

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HIGHLIGHTS

- Partial derivatives for heterogeneous coefficient SAR models are derived.
- Scalar summary measures proposed in the literature are not likely to work here.
- Observation-level marginal effects are proposed.
- Non-linear relationships between estimates and observation-level marginal effects arise as in probit.
- Spatial spill-out and spill-in effects can be quantified.

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ABSTRACT

We consider interpretation of estimates from the heterogeneous coefficient spatial autoregressive panel model of Aquaro et al. (2015) and derive partial derivatives (marginal effects) for this model, an issue not discussed in Aquaro et al. (2015). We show how these differ from a conventional spatial autoregressive panel model.

1. Introduction

Aquaro et al. (2015) make the observation that space–time panel data samples covering longer time spans are becoming increasingly prevalent. We let N denote the number of spatial units in the sample and T the number of time periods. Panel data sets with sufficiently large T allow us to exploit sample data along the time dimension to produce parameter estimates for all N spatial units. Allowing for heterogeneous coefficients for each spatial unit holds a natural appeal when contrasted with conventional static spatial panel models. The conventional static panel SAR model takes the matrix–vector form: $y = \psi(I_T \otimes W)y + (\iota_T \otimes \iota_N)\alpha + X\beta + \varepsilon$, where we ignore region-specific and time-specific fixed effects as they do not enter into our discussion. The NT-vector y is related to contemporaneous values from neighboring regions, $(I_T \otimes W)y$, with the scalar ψ reflecting the amount of spatial interaction between neighboring regions. The $NT \times K$ matrix X contains explanatory variables for each of the N regions over the T time periods, and the $K \times 1$ vector β are associated coefficients, and α a scalar intercept term. Disturbances in the $NT \times 1$ vector ε are typically assumed to be normally distributed with constant scalar variance and zero covariance (see Elhorst, 2014).

In conventional homogeneous models, parameters β describing the relationship between NT outcomes in the vector y and the $NT \times K$ matrix of regional characteristics X, are assumed the same for all regions and time periods in the sample. In addition, the scalar parameter ψ representing the level of spatial interaction between observed outcomes over time and space in the NT vector y and neighboring region outcomes in the NT spatial lag vector $(I_T \otimes W)y$,

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are assumed the same for all regions and time periods. Aquaro et al. (2015) argue there are a great many economic reasons to believe that the level of interaction between (say US states) differs greatly when considering patterns of spatial spillovers associated with things like tobacco consumption, house prices, labor market variables, etc. We should also note that some spatial data represent point-level rather than regional observations. For example, we may have spatial observations on firms such as gas stations, and wish to consider pricing behavior (price reaction functions) of these firms. Conventional/homogeneous models would allow us to estimate a single reaction function reflecting behavioral reactions of the average or typical firm. The heterogeneous coefficient model would allow us to estimate firm-level reactions, which seems desirable in cases where we are studying firm-level decisions.

Aquaro et al. (2015) provide theoretical results showing consistent asymptotic normality of quasi maximum likelihood (QML) estimates for a heterogeneous coefficients spatial autoregressive (HSAR) panel model specification that allows for variation in the level of spatial dependence/interaction ($\psi_i,\ i = 1, \dots, N$), variation in coefficients ($\alpha_i, \beta_i,\ i = 1, \dots, N$), and noise variances ($\sigma_i^2,\ i = 1, \dots, N$). They also explore small sample performance of their QML estimator using a Monte Carlo study that considers cases ranging over $N = 25$ to $N = 100$, and $T = 25$ to $T = 200$.

They do not discuss *interpretation of HSAR estimates*, which involves considering the partial derivatives of y with respect to changes in the K different explanatory variables in the matrix X (excluding any constant term). In Section 2 we present the heterogeneous SAR model. We discuss interpretative considerations for these models and contrast these with conventional homogeneous coefficient SAR models in Section 3.1. We set forth the partial derivatives (marginal effects) for these models in Section 3.2 which serve as the basis for inference regarding the impact of changes in explanatory variables on outcomes.

2. The HSAR model

The heterogeneous SAR model (which we label HSAR hereafter) can be written as in (1), where w_{ij} represents the i, jth element of a normalized spatial weight matrix with $w_{ii} = 0$.[1]

$$y_{it} = \alpha_i + \psi_i \sum_{j=1}^{N} w_{ij} y_{jt} + \sum_{k=1}^{K} \beta_i^k x_{it}^k + \varepsilon_{it},$$
$$i = 1, 2, \dots, N,\ t = 1, 2, \dots, T. \tag{1}$$

The set of K explanatory variables x_{it}^k are assumed exogenous, and we require that covariance matrices $E(x_{it}^k x_{jt}^{k'})$, $\forall\, i, j, k$ are time-invariant and finite as well as non-singular. The requirement of time-invariance arises because we are using the time dimension of the sample data to estimate parameters for each regional unit, $i = 1, \dots, N$. The disturbances are assumed distributed independently, and for our purposes we can assume these follow independent normal distributions with a different variance (σ_i^2) for each observation.

The HSAR model can be written in matrix notation as shown in (2) by stacking regional units,

$$y_t = \alpha + \Psi W y_t + \sum_{k=1}^{K} B^k x_t^k + \varepsilon_t \tag{2}$$

where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_N)'$, $\Psi = \mathrm{diag}(\psi)$, $\psi = (\psi_1, \psi_2, \dots, \psi_N)$, $W = w_{ij},\ i, j = 1, \dots, N$, $B^k = \mathrm{diag}(\beta_1^k, \beta_2^k, \dots, \beta_N^k)$, $x_t^k = (x_{1t}^k, x_{2t}^k \dots, x_{Nt}^k)'$, $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{Nt})'$, with $\varepsilon_t \sim N[\mathrm{diag}(0), \mathrm{diag}(\Sigma)]$, $\Sigma = (\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2)$. That is, we assume disturbances that are normally distributed with zero mean and zero covariance, but allow for separate variances for each region.

The data generating process for the HSAR model can be written as:

$$y_t = (I_N - \Psi W)^{-1} \left(\alpha + \sum_{k=1}^{K} B^k x_t^k + \varepsilon_t \right). \tag{3}$$

3. A comparison of homogeneous and heterogeneous model effects estimates

In Section 3.1 we review interpretation of the homogeneous coefficient SAR model, which is contrasted with our proposed interpretation of the heterogeneous coefficient SAR model in Section 3.2.

3.1. Interpreting homogeneous coefficient SAR models

In the case of homogeneous static panel data models, Elhorst (2014) follows LeSage and Pace (2009) proposing an average of the main diagonal elements of the $N \times N$ matrix of partial derivatives for this model shown in (4) as a *scalar summary measure* of own-partial derivatives that LeSage and Pace (2009) label *direct effects*. This simplifies the task of interpreting estimates from the model, which take the form of an $N \times N$ matrix for each of the K explanatory variables. LeSage and Pace (2009) also propose a scalar summary measure of the *indirect effects* (spatial spillover) impacts based on the cumulative sum of the off-diagonal elements from each row, averaged over all rows. A scalar summary measure of the total impact of a change in regional outcomes arising from changes in the kth regional characteristic is the sum of the scalar direct plus indirect effects estimates. In the case of the homogeneous coefficient models where all $\psi_i = \psi$ and all $\beta_i^k = \beta^k$, this approach holds intuitive appeal.

The motivation for this approach is based on the $N \times N$ matrix of partial derivatives for the homogeneous SAR model:

$$\partial y / \partial X^{k'} = \begin{pmatrix} \partial y_1/\partial x_1^k & \partial y_1/\partial x_2^k & \cdots & \partial y_1/\partial x_N^k \\ \partial y_2/\partial x_1^k & \partial y_2/\partial x_2^k & \cdots & \partial y_2/\partial x_N^k \\ \vdots & \vdots & \ddots & \vdots \\ \partial y_N/\partial x_1^k & \partial y_N/\partial x_2^k & \cdots & \partial y_N/\partial x_N^k \end{pmatrix}$$
$$= (I_N - \psi W)^{-1} I_N \beta^k \tag{4}$$

where Elhorst (2014) notes that this expression arises from recognizing that the coefficients ψ, β do not change over the time periods of the panel. Main diagonal elements of the matrix $(I_N - \psi W)^{-1} I_N \beta^k$ reflect own-partial derivatives and off-diagonal elements represent cross-partial derivatives.

Although LeSage and Pace (2009) do not recommend reporting *observation-level* effects estimates, one could calculate these for the homogeneous coefficients SAR model using the $N \times 1$ diagonal elements of the matrix $(I_N - \psi W)^{-1} I_N \beta^k$ as observation-level direct effects, and the cumulative sum of off-diagonal rows (or columns) of this matrix as observation-level indirect effects. Since this approach would correspond most closely to observation-level effects estimates that we propose in the next section for the heterogeneous SAR model, we enumerate properties of these observation-level direct and indirect effects estimates for the case of the homogeneous SAR specification.

(1) For spatial weight matrices based on some fixed number m of equally weighted neighbors for all observations, there is no variation in the diagonal elements of the $N \times N$ matrix $(I_N - \psi W)^{-1} I_N \beta^k$, and cumulative sums of off-diagonal row- or column-elements corresponding to observation-level direct and indirect effects.

[1] We will have more to say about specific normalization schemes later.