Endogenous evolution of heterogeneous consumers preferences: Multistability and coexistence between groups

Ahmad Naimzada [a,1], Marina Pireddu [b,*]

[a] *Department of Economics, Management and Statistics, University of Milano-Bicocca, U6 Building, Piazza dell'Ateneo Nuovo 1, 20126 Milano, Italy*
[b] *Department of Mathematics and Applications, University of Milano-Bicocca, U5 Building, Via Cozzi 55, 20125 Milano, Italy*

HIGHLIGHTS

- We propose an evolutive model in an exchange economy setting.
- Agents are heterogeneous in the structure of the preferences.
- The share updating mechanism is non-monotone in the calorie intake.
- We find multistability phenomena involving equilibria with heterogeneous agents.

ABSTRACT

We propose an exchange economy evolutionary model with agents heterogeneous in the structure of preferences. Assuming that the share updating mechanism is non-monotone in the calorie intake, we find multistability phenomena involving equilibria characterized by the coexistence of heterogeneous agents.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

Chang and Stauber (2009) propose a model on the evolution of the population shares in an exchange economy setting, in which it is assumed that there are two groups of agents characterized by a different structure of the preferences. Indeed, the weights assigned to the two consumption goods in the Cobb–Douglas utility functions do not coincide across groups. The mechanism according to which shares are updated in Chang and Stauber (2009) is monotone in the calorie intake: the larger such intake by a group,

the more that group share increases. The results in that paper concern the existence and local stability of trivial and nontrivial market stationary equilibria, where we call equilibria trivial if they are not characterized by the coexistence between the two groups of agents, as one of the two groups totally prevails on the other and remains alone. Chang and Stauber (2009) find at most one nontrivial market stationary equilibrium, which when exists is stable, and two trivial equilibria.

According to Chang and Stauber (2009), a monotone population growth rate is suitable to represent the long-run centuries-old trend, as the diet of a population group affects its long-term survival. On the other hand, in that paper it is remarked that it would be interesting to consider more general biological payoff functions. In fact, supported by the empirical literature, we believe that a biological payoff function monotonically increasing in the calorie intake is not well suited to describe

---

* Corresponding author. Tel.: +39 0264485767; fax: +39 0264485705.
E-mail addresses: ahmad.naimzada@unimib.it (A. Naimzada),
marina.pireddu@unimib.it (M. Pireddu).
[1] Tel.: +39 0264485813; fax: +39 0264483085.

the framework of contemporary developed countries. Namely, according to Ponthiere (2011), monotone survival functions do not fit aggregate data (cf. Fig. 1 therein); moreover, a broad epidemiological literature (see e.g. Adams et al., 2006; Bender et al., 1998; Fontaine et al., 2003 and Solomon and Manson, 1997) has shown the negative effects of overconsumption on health and survival, emphasizing a non-monotone relationship between the corpulence, measured by BMI (Body-Mass Index, i.e., the weight in kilograms divided by the square of the height in meters), and mortality risks. Such non-monotone relationship is clearly represented in the (BMI, mortality)-plane by Waaler (1984)'s U-shaped curves, which have been commented by Fogel (1994) from an intertemporal viewpoint. The relevance of those findings gave rise to the so-called "economics of obesity" (see the survey by Philipson and Posner, 2008).

In light of the above observations, we then aim to reconsider the model in Chang and Stauber (2009), replacing the monotone population growth rate assumed therein with a bell-shaped map, increasing with the calorie intake up to a certain threshold value, above which it becomes decreasing. Even with such a crucial change, we still obtain a one-dimensional continuous-time dynamical system, for which, in addition to the three equilibria in Chang and Stauber (2009), we find (up to) two additional nontrivial market stationary equilibria. Moreover, the (possibly existing) nontrivial equilibrium found in Chang and Stauber (2009) may become unstable in our context, and also the two trivial equilibria may have different dynamic behaviors with respect to Chang and Stauber (2009). We perform a qualitative bifurcation analysis on varying the parameter describing the threshold value at which the growth rate becomes decreasing. In particular, differently from the framework in Chang and Stauber (2009), our setting displays multistability phenomena, characterized by the presence of multiple, trivial and nontrivial, locally stable market stationary equilibria.

We remark that multistability may be considered as a source of richness for the framework under analysis because, other parameters being equal, i.e., under the same institutional, cultural and social conditions, it allows to explain different trajectories and evolutionary paths. The initial conditions, leading to the various attractors, represent indeed a summary of the past history, which in the presence of multistability phenomena does matter in determining the evolution of the system. Such property, in the literature on complex systems, is also called "path-dependence" (see Arthur, 1994).

Moreover, in the specific context we deal with, the presence of multiple equilibria well represents the variety of historical experiences across different countries in relation to the approach they adopt towards food, diet and consequently towards obesity (consider e.g., according to Philipson and Posner, 2008, the different scenarios in the US and in the Mediterranean countries).

Finally, we stress that in our model we also analyzed the setting in which the two groups of agents may differ in the threshold level, above which an increase in the calorie intake becomes harmful rather than beneficial. Such scenario can describe for instance the case in which the two groups of agents differ in the amount of sports they play: inactive people need a lower calorie intake than athletes. Since the possible dynamic scenarios we found were similar to those obtained in the setting in which the two threshold levels coincide, we preferred to confine ourselves to the simpler framework, which also allows a deeper analytical treatment.

2. The model

We start our discussion recalling the framework in Chang and Stauber (2009), where the authors consider a continuous-time model describing an exchange economy with a continuum of agents, which may be of type α or of type β. There are two consumption goods, x and y, and agent preferences are described by Cobb–Douglas utility functions, i.e., $U_i(x, y) = x^i y^{1-i}$, for $i \in \{\alpha, \beta\}$, with $0 < \beta < \alpha < 1$. Both kinds of agents have the same endowments of the two goods, denoted respectively by w_x and w_y. The analysis is performed in terms of the relative price $p(t) = p_y(t)/p_x(t)$, where $p_x(t)$ and $p_y(t)$ are the prices at time t for goods x and y, respectively. The size of the population of kind α (β) at time t is denoted by $A(t)$ ($B(t)$). The calorie intake $K_i(t)$ of an agent of type $i \in \{\alpha, \beta\}$ at time t is given by a linear combination of the units $x_i(t)$ and $y_i(t)$ of goods x and y he consumes, weighted respectively with the calories that each agent derives from the consumption of a unit of good x and of good y, i.e., $K_i(t) = c_x x_i(t) + c_y y_i(t)$. In Chang and Stauber (2009), denoting by $\overline{K}$ the calorie subsistence level, the growth rate of the population of type i is then assumed to be

$$K_i(t) - \overline{K}, \tag{2.1}$$

so that the evolution of the two groups of consumers is described by the following system

$$\begin{cases} \dfrac{dA(t)}{dt} = (K_\alpha(t) - \overline{K})A(t) \\ \dfrac{dB(t)}{dt} = (K_\beta(t) - \overline{K})B(t). \end{cases} \tag{2.2}$$

Introducing the normalized variables $a(t) = (A(t))/(A(t) + B(t))$ and $b(t) = (B(t))/(A(t) + B(t))$, describing the fractions of the population composed by the agents of type α and β, respectively, and noticing that $b(t) = 1 - a(t)$, System (2.2) becomes equivalent to

$$\frac{da(t)}{dt} = (K_\alpha(t) - K_\beta(t))a(t)(1 - a(t)). \tag{2.3}$$

The (normalized) price at which agents exchange their endowments is determined by solving the consumer maximization problem and using a market clearing condition. According to Chang and Stauber (2009), the market equilibrium price, i.e., the price that clears the market, is then given by

$$p^*(t) = \frac{\left[1 - \big(a(t)\alpha + (1 - a(t))\beta\big)\right]w_x}{\big(a(t)\alpha + (1 - a(t))\beta\big)w_y} \tag{2.4}$$

and the consumer equilibrium quantities of the two goods for agent of type $i \in \{\alpha, \beta\}$, compatible with the market equilibrium, are

$$x_i^*(t) = i(w_x + p^*(t)w_y) = \frac{iw_x}{a(t)\alpha + (1 - a(t))\beta},$$
$$y_i^*(t) = (1 - i)\left(\frac{w_x}{p^*(t)} + w_y\right) = \frac{(1 - i)w_y}{1 - (a(t)\alpha + (1 - a(t))\beta)}. \tag{2.5}$$

Hence, (2.3) can be rewritten as

$$\frac{da(t)}{dt} = (\alpha - \beta)a(t)(1 - a(t))$$
$$\times \left(\frac{c_x w_x}{a(t)\alpha + (1 - a(t))\beta} - \frac{c_y w_y}{1 - a(t)\alpha - (1 - a(t))\beta}\right). \tag{2.6}$$

The market stationary equilibria, at which for every t the population shares, and thus also the market equilibrium price and the consumer equilibrium quantities, are constant, will be called trivial if they are not characterized by the coexistence between the two groups of agents, and nontrivial otherwise. In addition to the trivial market stationary equilibria $a = 0$ and $a = 1$, a nontrivial market stationary equilibrium is given by $a = a^*$, with

$$a^* = \frac{(1 - \beta)c_x w_x - \beta c_y w_y}{(\alpha - \beta)(c_x w_x + c_y w_y)}, \tag{2.7}$$