Economics Letters 142 (2016) 74–77

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Tullock contests may be revenue superior to auctions in a symmetric setting

Alexander Matros [a,b], Alex Possajennikov [c,*]

[a] Darla Moore School of Business, University of South Carolina, USA
[b] Lancaster University Management School, Lancaster, UK
[c] School of Economics, University of Nottingham, University Park, Nottingham NG7 2RD, UK

HIGHLIGHTS

- A symmetric two-player common-value setting with signals is considered.
- Equilibrium revenue of the Tullock contest is compared to that of the auctions.
- For some parameters, revenue may be higher in the contest than in the auctions.

ARTICLE INFO

Article history:
Received 4 November 2015
Received in revised form
27 February 2016
Accepted 4 March 2016
Available online 11 March 2016

JEL classification:
D82
D44

Keywords:
Common value
Contests
Auctions

ABSTRACT

We consider a symmetric two-player common-value setting where each player gets a private signal about the object value. We show that for some parameter values the equilibrium revenue can be higher in a Tullock contest than in the standard auctions.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

Economics literature often considers the problem of allocating an indivisible object among individuals. Typical mechanisms for such an allocation are (deterministic) auctions, but an object can also be allocated using a stochastic mechanism. One of the most common stochastic mechanisms is a Tullock contest (Tullock, 1980).

It is often the case in a symmetric setting that the seller's revenue is higher in the standard auctions than in a contest. For example, if an object with a common and known value is allocated between two players, the seller's revenue is equal to the object's value in a standard auction, while it is only one-half of the value in the simplest Tullock contest ("lottery", where the probability of winning the object for a player is the ratio of her bid to the sum of all bids).[1] On the other hand, Fang (2002) and Epstein et al. (2013) have shown that a contest can achieve a higher revenue than an all-pay auction in asymmetric settings; Franke et al. (2014) further argue that the presence of more than two players increases the chances of contests to achieve a higher revenue. In this paper, we show that a Tullock contest can generate a higher expected revenue even in an ex-ante symmetric setting with two players.

We consider a common-value setting where each player gets a private signal about the object value. Very little is known about revenue comparison of auctions and contests in situations where players have private information, because these situations are

* Corresponding author. Tel.: +44 115 9515461; fax: +44 115 9514159.
E-mail addresses: alexander.matros@gmail.com (A. Matros), alex.possajennikov@nottingham.ac.uk (A. Possajennikov).

[1] Although in a different setting, Wärneryd (2012, p. 278) also writes that "[f]rom the standpoint of a seller offering a good in an auction a perfectly discriminating mechanism is optimal …".

http://dx.doi.org/10.1016/j.econlet.2016.03.003
0165-1765/© 2016 Elsevier B.V. All rights reserved.

typically intractable for contests. In this paper we analyze the Tullock contest and the standard (first-price, second-price and all-pay) auction mechanisms in a symmetric two-player common-value setting where both contest and auction equilibria could be explicitly derived and compared. In our setting, each player receives an independent signal and the common value of the object is an increasing function of both players' signals. The setting is similar to Klemperer's (1998) "wallet game", in which the common value is the sum of the signals of the two players.[2]

In this setting, we derive the equilibrium for the Tullock contest. For the standard auctions, we use, with minor modifications, the available results from the literature. Although generally there is no clear ranking of auctions in terms of revenue in common-value settings (see Milgrom and Weber, 1982 and Malueg and Orzach, 2009, 2012), in our setting with independent signals the seller's revenue is the same in all three auction mechanisms. Comparing this revenue with the one from the contest, we identify parameter values for which the expected revenue in the contest is higher than in any of the auctions.

2. Setting: private signals and common value

There is an object for sale and there are two risk-neutral buyers. Each of the two buyers gets a private signal about the value of the object: Buyer 1 receives signal s_1 and Buyer 2 gets signal s_2. Suppose that the signals have the following structure: each signal is either H (high) with probability $p \in [0,1]$, or L (low, $L < H$) with probability $1-p$, independently of the other signal. That is,

$$s_i = \begin{cases} H, & \text{with probability } p, \\ L, & \text{with probability } 1-p. \end{cases}$$

The common value of the object is a function of the two private signals, $v = g(s_1, s_2)$. This function satisfies $g(L,L) = 0, g(L,H) = g(H,L) = V > 0$ and $g(H,H) = (1+\alpha)V$, for $\alpha \geq 0$. The parameter α captures possible nonlinearity or complementarity in the signals.

Two risk-neutral buyers make bids and the object is allocated to one of them according to some mechanism. We first consider the mechanism where the object is allocated according to the Tullock contest and then the standard (all-pay, first-price, and second-price) auction mechanisms.

2.1. Tullock contest

In the contest, if the bids of the two buyers are x_i and x_j, then buyer i wins the object with probability $x_i/(x_i + x_j), j \neq i$. A pure strategy $x^i = (x^i_L, x^i_H)$ of buyer i consists of two bids, x^i_L if her signal is L and x^i_H if her signal is H. The expected payoff of buyer i, conditional on the received signal, is

$$\begin{aligned} u_i(x^i_L, x^j | s_i = L) &= (1-p)\frac{x^i_L}{x^i_L + x^j_L} \times 0 \\ &\quad + p\frac{x^i_L}{x^i_L + x^j_H} \times V - x^i_L; \quad (1) \\ u_i(x^i_H, x^j | s_i = H) &= (1-p)\frac{x^i_H}{x^i_H + x^j_L} \times V \\ &\quad + p\frac{x^i_H}{x^i_H + x^j_H} \times (1+\alpha)V - x^i_H. \quad (2) \end{aligned}$$

We denote this game by $\mathcal{L}$.

[2] Klemperer considers only the second-price auction in his paper.

Proposition 1. *In the unique symmetric pure-strategy equilibrium of the contest game $\mathcal{L}$, equilibrium bids $x^i_L = x^j_L = x_L$ and $x^i_H = x^j_H = x_H$ are*

$$x_L = \begin{cases} \frac{1}{4}pV\left(1-p+D(p,\alpha)\right)\left(1+p-D(p,\alpha)\right), \\ \quad \text{if } 0 \leq \alpha \leq 3, \\ 0, \quad \text{if } \alpha > 3, \end{cases} \quad (3)$$

and

$$x_H = \begin{cases} \frac{1}{4}pV\left(1-p+D(p,\alpha)\right)^2, & \text{if } 0 \leq \alpha \leq 3, \\ \frac{1}{4}p(1+\alpha)V, & \text{if } \alpha > 3, \end{cases} \quad (4)$$

where

$$D(p,\alpha) = \sqrt{1-p+p^2+\alpha p}.$$

Proof. Buyer i maximizes her expected payoffs (1)–(2). Then, the first order conditions are

$$p\frac{x^j_H}{(x^i_L + x^j_H)^2}V - 1 = 0 \quad (5)$$

$$(1-p)\frac{x^j_L}{(x^i_H + x^j_L)^2}V + p\frac{x^j_H}{(x^i_H + x^j_H)^2}(1+\alpha)V - 1 = 0. \quad (6)$$

The second order conditions are satisfied as the left-hand sides of the above expressions are decreasing in x^i_L and x^i_H respectively.

In a symmetric equilibrium, $x^j_L = x^i_L = x_L$ and $x^j_H = x^i_H = x_H$. From Eq. (5), $x_L = \sqrt{pVx_H} - x_H$. Eq. (6) becomes

$$-4x_H + 4(1-p)\sqrt{pV}\sqrt{x_H} + (1+\alpha)p^2V = 0.$$

The unique positive solution of this quadratic equation is

$$x_H = \frac{1}{4}pV(1-p+D(p,\alpha))^2,$$

where $D(p,\alpha) = \sqrt{1-p+p^2+\alpha p}$. Then,

$$x_L = \frac{1}{4}pV\left(1-p+D(p,\alpha)\right)\left(1+p-D(p,\alpha)\right).$$

This last expression becomes negative for $\alpha > 3$. Thus, the previous derivations hold for $\alpha \leq 3$. For $\alpha > 3$, $x^i_L = x^j_L = x_L = 0$ and $x^i_H = x^j_H = x_H = p(1+\alpha)V/4$. ■

The ex-ante expected revenue in the equilibrium of the contest game is

$$\begin{aligned} \pi^{\mathcal{L}}(p,\alpha) &= (1-p)^2 \cdot 2x_L + 2(1-p)p \cdot (x_L + x_H) \\ &\quad + p^2 \cdot 2x_H = 2\left((1-p)x_L + px_H\right). \quad (7) \end{aligned}$$

From Proposition 1, we get

Proposition 2. *In the equilibrium of the contest game $\mathcal{L}$, the ex-ante expected revenue is*

$$\pi^{\mathcal{L}}(p,\alpha) = \begin{cases} \frac{1}{2}pV\left(1-p+D(p,\alpha)\right)\left(1+p-2p^2 \right. \\ \quad \left. +(2p-1)D(p,\alpha)\right), \quad \text{if } 0 \leq \alpha \leq 3, \\ \frac{1}{2}p^2(1+\alpha)V, \quad \text{if } \alpha > 3. \end{cases} \quad (8)$$

2.2. Standard auctions

In an auction, the highest bid wins the object for sure. We denote by $\mathcal{A}$ any standard (all-pay, first-price, second-price) auction.

Download English Version:

https://daneshyari.com/en/article/5058265

Download Persian Version:

https://daneshyari.com/article/5058265

Daneshyari.com