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Advertising intensity and welfare in an equilibrium search model

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HIGHLIGHTS

- I analyze a consumer search model where firms can advertise by announcing price.
- I examine the firm advertising level relative to that of a social planner.
- Firms over-advertise if search costs are sufficiently low.
- Firms under-advertise if search costs are sufficiently high.

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ABSTRACT

I analyze an equilibrium search model in a duopoly setting with bilateral heterogeneities in production and search costs in which firms can advertise by announcing price. I compare the market advertising level to the socially optimal level, where I find that costly search can improve welfare and that firms may under- or over-advertise relative to the social optimum depending on the costs of search. The results suggest that, in markets with sufficiently low search costs, firms are likely over-advertising relative to the socially optimal level, and vice versa for markets with sufficiently high search costs.

1. Introduction

Imperfect price information is a fundamental aspect of any consumer search model. Avenues that can improve this information, such as advertising, therefore have a natural role in allowing consumers to refine their knowledge of prices as they receive new information. In this article, I study an equilibrium search model in a duopoly setting and introduce an advertising technology by which firms can inform consumers of their price. I ask, if consumers engage in optimal search, will firms over- or under-advertise relative to the socially optimal level? This question is relevant for a range of markets where consumers have incomplete information, can pursue costly search to find a lower price or a "better" product, and where firms can engage in some form of informative advertising.

With a simplified version of the search models developed in Carlson and McAfee (1983), Benabou (1993), Robert and Stahl (1993), Rob (1985), and others, this article offers two important contributions to the literature: (1) Costly search can have positive welfare effects when firms advertise; and (2) relative to the socially optimal advertising level, firms over-advertise when search costs are sufficiently low and under-advertise otherwise. These results illustrate the importance of the interaction between consumers and firms. Through search, consumers have their own avenue to gain information, and through advertising, firms can improve the search process. But advertising is costly and introduces secondary price effects. Search may therefore be socially beneficial in the sense that the planner allows costly consumer search. Understanding the relationship (and the welfare effects) of search and advertising is an important issue since, by definition, informative advertising only exists inasmuch as consumers have incomplete information. The true welfare effects of informative advertising therefore hinge on consumers' ability to gain information independently, and search is one important way to model this ability. Previous advertising and search models

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have not formally studied this interaction *vis à vis* welfare (Butters, 1977; Stegeman, 1991; Robert and Stahl, 1993; Janssen and Non, 2008).

2. The model

Market size is normalized to one, and consumers have heterogeneous search costs with cdf $Q(s)$, pdf $q(s)$, and full support on $[0, 1]$. I assume that $Q(0) = 0$, $q(s)$ is twice continuously differentiable, and that $q(s)$ is bounded with $q_{\max} = \max_{s\in[0,1]} q(s)$ and $q_{\min} = \min_{s\in[0,1]} q(s)$. Consumers are perfectly informed as to the distribution of prices but do not know which firms offer which price (Carlson and McAfee, 1983; Benabou, 1993), and inelastically demand one unit up to some maximum price, $p_{\max}$. Buyers enter the market with a free initial search but must pay their search cost to visit another firm.[1]

There are two firms, each producing identical goods with heterogeneous costs of production. The low cost firm has marginal cost normalized to zero, while the high cost firm has constant marginal cost $c > 0$. Denote the low and high cost firms' prices by p_L and p_H respectively. Both firms can advertise their price to a fraction of the market at some constant marginal cost $A > 0$. Since the distribution of prices is known, any consumer receiving an advertisement is then perfectly informed of prices. I denote the level of advertising by $x \in [0, 1]$ and assume that the advertising firm is required to charge the price advertised. Given x, denote the proportion of uninformed consumers by $f(x)$, where $f(x)$ satisfies $f' < 0, f'' \geq 0, f(0) = 1$, and $f(1) = 0$. The proportion of informed consumers is then $1 - f(x)$.

With two firms, the consumer's search decision is based on

$$p_H - s \geq p_L, \tag{2.1}$$

which yields the critical search value $\hat{s} = p_H - p_L$—below which consumers search find the low cost firm and above which consumers purchase from whichever firm they randomly choose, i.e., for $s \geq \hat{s}$, consumers are inactive searchers. In the absence of advertising, this model is similar to the duopoly search model examined in Rauh (2004).

Both firms take as given consumer behavior described above and play the subsequent game with prices and advertising as strategic variables. Denoting the low cost firm by L and the high cost firm by H, each firm faces the following demands:

$$q_L = \left[1 - \frac{1}{2}f(x)(1 - Q(\hat{s}))\right]; \tag{2.2}$$

$$q_H = \frac{1}{2}f(x)(1 - Q(\hat{s})). \tag{2.3}$$

Eq. (2.3) derives from some proportion of consumers being informed of the low price via advertising, leaving $f(x)$ uninformed. Of these, half randomly select the high cost firm, and some portion $Q(\hat{s})$ are active searchers with sufficiently low search costs so that they never pay the high price. The remaining consumers go to the low cost firm, which yields (2.2). Under reasonable assumptions detailed in Appendix A, a (stable) Nash equilibrium of this game exists wherein the low cost firm optimally prices below the high cost firm. As a result, the high cost firm optimally chooses not to advertise.

The resulting first order conditions are[2]:

$$\frac{\partial \pi_L}{\partial p_L} = \left[1 - \frac{1}{2}f(x)(1 - Q(\hat{s}))\right] - \frac{1}{2}q(\hat{s})f(x)p_L = 0; \tag{2.4}$$

$$\frac{\partial \pi_L}{\partial x} = -\frac{1}{2}f'(x)(1 - Q(\hat{s}))p_L - A = 0; \tag{2.5}$$

$$\frac{\partial \pi_H}{\partial p_H} = \frac{1}{2}f(x)(1 - Q(\hat{s})) - \frac{1}{2}f(x)q(\hat{s})(p_H - c) = 0. \tag{2.6}$$

3. Welfare and advertising intensity

In this model, advertising plays a purely informational role in announcing the true price of the low cost firm and thus implicitly doing so for the high cost firm; however, even in this simplified framework, welfare effects of advertising are unclear due to inherent differences in the objectives of the social planner and the advertising firm. To fully characterize when and how these differences might lead to over- or under-advertising relative to a planner, I study the firm's advertising level relative to the level chosen by a social planner.

Assuming an interior solution, the social planner solves

$$\max_{x\in[0,1]} \bar{u} - Ax - \frac{1}{2}f(x)\int_0^{\hat{s}} sq(s)ds - \frac{1}{2}f(x)(1 - Q(\hat{s}))c, \tag{3.1}$$

subject to

$$\frac{\partial \pi_L}{\partial p_L} = \left[1 - \frac{1}{2}f(x)(1 - Q(\hat{s}))\right] - \frac{1}{2}f(x)q(\hat{s})p_L = 0 \tag{3.2}$$

$$\frac{\partial \pi_H}{\partial p_H} = -q(\hat{s})(p_H - c) + 1 - Q(\hat{s}) = 0, \tag{3.3}$$

where $\hat{s} = p_H - p_L$.

The planner advertises essentially for two reasons: (1) so that consumers reach the low cost firm on their first attempt and do not pay additional search costs; and (2) to save the cost of production incurred by the high cost firm. The constraints impose the firms' pricing conditions on the planner's problem, therefore focusing on a structural second-best where the planner chooses advertising at prices consistent with firm behavior.[3]

To solve the planner's problem, I first solve the constraints implicitly for $\hat{s}(x)$ and substitute this into the planner's objective function.[4] Denoting the mean search cost consumer by μ, the following proposition summarizes the conditions in which the firm over or under-advertises relative to the planner:

Proposition 1. *For the search cost pdf, $q(s)$, satisfying*

$$\frac{q'(\hat{s})}{q(\hat{s})} \in \left(-\frac{2}{(p_H - c)}, \frac{2}{p_L}\right), \quad \textit{and}$$

$$\frac{q''(\hat{s})}{q(\hat{s})} \leq \frac{-2f''(x)}{f'(x)(2p_{\max} - c)c} - \frac{2}{(p_{\max} - c)c},$$

and the advertising technology, $f(x)$, satisfying

$$\frac{f'(x)}{f(x)} > -\frac{1}{2}f(x)q(\hat{s}), \quad \textit{and}$$

$$\frac{f''(x)}{f'(x)} < -\frac{1}{1 - Q(\hat{s})}\left[\frac{2}{p_L f(x)} + q(\hat{s})\right],$$

(i) *there exist some $\bar{c}$, $\underline{A}$, and $p_{\max}$ such that the duopolistic advertising level always exceeds the socially optimal level for all $c \geq \bar{c}$, all $A \leq \underline{A}$, or all $p_{\max} \leq \frac{2}{q_{\max}} - \mu$;*

[1] A free first search is equivalent to assuming that all buyers will make a purchase. This assumption therefore avoids keeping track of which buyers stay in the market. See Janssen et al. (2005) for a relaxation of this assumption.

[2] See Appendix B for discussion of comparative statics.

[3] See Vives (2001) Chapter 6 for a similar approach with product differentiation.

[4] Under relatively mild conditions on the search cost distribution, it can be shown that a unique solution to the planner's problem exists. See Appendix C for details.