



The impossibility of extending random dictatorship to weak preferences



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HIGHLIGHTS

- Random dictatorship (RD) is efficient and strategyproof for strict preferences.
- It is the only social decision scheme satisfying both properties (Gibbard, 1977).
- We show that no extension of RD to weak preferences satisfies both properties.
- This holds even when significantly weakening the required degree of strategyproofness.

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ABSTRACT

Random dictatorship has been characterized as the only social decision scheme that satisfies efficiency and strategyproofness when individual preferences are strict. We show that no extension of random dictatorship to weak preferences satisfies these properties, even when significantly weakening the required degree of strategyproofness.

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1. Introduction

One of the most celebrated results in microeconomic theory is the Gibbard–Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975), which states that every strategyproof and Pareto-optimal social choice function is a dictatorship. However, the theorem crucially relies on the assumption that outcomes are deterministic. Gibbard (1977) later considered social decision schemes, i.e., social choice functions that return lotteries over the alternatives, and showed that the class of strategyproof and *ex post* efficient functions extends to all random dictatorships. This class

contains a unique rule that treats all agents equally: the uniform random dictatorship, henceforth *random dictatorship* (RD), where an agent is chosen uniformly at random and his favorite alternative is implemented as the social choice. Gibbard's notion of strategyproofness is based on *stochastic dominance* and requires that there is no expected utility representation consistent with the voters' ordinal preferences such that a voter can obtain more utility by misrepresenting his preferences. Another implicit assumption in Gibbard's theorem is the anti-symmetry of individual preferences. Characterizations of strategyproof social decision schemes for the case when agents are allowed to express indifference have also been explored. In the context of cardinal decision schemes, Dutta et al. (2007) characterize RD for the domain in which each agent has a unique top choice. For arbitrary weak preferences, Hylland (1980) and Nandeibam (2013) show that the only reasonable strategyproof social decision schemes are weak random dictatorships.

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We refer to Nandeibam (2013) for a discussion of these results. Perhaps the best-known generalization of *RD* to weak preferences is *random serial dictatorship (RSD)* where a permutation of agents is chosen uniformly at random and agents narrow down the set of alternatives in that order to their most preferred alternatives among the remaining alternatives. *RSD* is also *ex post* efficient and strategyproof with respect to stochastic dominance. However, in contrast to *RD* it is not efficient with respect to stochastic dominance, i.e., there might be a lottery that yields more expected utility for all agents. This failure of efficiency was first observed by Bogomolnaia and Moulin (2001) in the context of random assignment.¹ We show that this is not a weakness specific to *RSD* but in fact all fair generalizations of *RD* violate either efficiency or strategyproofness, even when significantly weakening the required degree of strategyproofness.²

2. Preliminaries

Let $N = \{1, \dots, n\}$ be a set of agents with preferences over a finite set A with $|A| = m$. The preferences of agent $i \in N$ are represented by a complete, reflexive, and transitive preference relation $\succsim_i \subseteq A \times A$. The set of all preference relations will be denoted by $\mathcal{R}$. In accordance with conventional notation, we write $\succ_i$ for the strict part of $\succsim_i$, i.e., $x \succ_i y$ if $x \succsim_i y$ but not $y \succsim_i x$ and $\sim_i$ for the indifference part of $\succsim_i$, i.e., $x \sim_i y$ if $x \succsim_i y$ and $y \succsim_i x$. We will compactly represent a preference relation as a comma-separated list with all alternatives among which an agent is indifferent placed in a set. For example $a \succ_i b \sim_i c$ will be written as $\succsim_i: a, \{b, c\}$. A preference relation $\succsim_i$ is *strict* if $x \succ y$ or $y \succ x$ for all distinct alternatives x, y . A *preference profile* $R = (\succsim_1, \dots, \succsim_n)$ is an n -tuple containing a preference relation $\succsim_i$ for each agent $i \in N$. The set of all preference profiles is thus given by $\mathcal{R}^n$. By R_{-i} we denote the preference profile obtained from R by removing the preference relation of agent i , i.e., $R_{-i} = R \setminus \{(i, \succsim_i)\}$.

Let furthermore $\Delta(A)$ denote the set of all *lotteries* (or *probability distributions*) over A and, for a given lottery $p \in \Delta(A)$, $p(x)$ denote the probability that p assigns to alternative x . Lotteries will be written as convex combinations of alternatives, e.g., $1/2 a + 1/2 b$ denotes the lottery p with $p(a) = p(b) = 1/2$.

Our central object of study are *social decision schemes*, i.e., functions that map the individual preferences of the agents to a lottery over alternatives. Formally, a social decision scheme (SDS) is a function $f: \mathcal{R}^n \rightarrow \Delta(A)$. A minimal fairness condition for SDSs is *anonymity*, which requires that $f(R) = f(R')$ for all $R, R' \in \mathcal{R}^n$ and permutations $\pi: N \rightarrow N$ such that $\succsim'_i = \succsim_{\pi(i)}$ for all $i \in N$. Another fairness requirement is *neutrality*. For a permutation of alternatives σ and a preference relation $\succsim_i$, $\sigma(x) \succsim_i^\sigma \sigma(y)$ if and only if $x \succsim_i y$. Then, an SDS f is *neutral* if $f(R)(x) = f(R^\sigma)(\sigma(x))$ for all $R \in \mathcal{R}^n$, $x \in A$, and permutations $\sigma: A \rightarrow A$.

Two well-studied SDSs are *Random Dictatorship (RD)* and *Random Serial Dictatorship (RSD)*. *RD* is defined when all agents have a unique favorite alternative. This includes the domain of strict preferences as a subclass. The lottery returned by *RD* is

obtained by choosing an agent uniformly at random and returning that agent's favorite alternative. *RSD* is an extension of *RD* to the full domain of preferences. *RSD* operates by first choosing a permutation of the agents uniformly at random. Starting with the set of all alternatives, it then asks each agent in the order of the permutation to choose his favorite alternative(s) among the remaining alternatives. If more than one alternative remains after taking the preferences of all agents into account, *RSD* uniformly randomizes over those alternatives. Formally, we obtain the following recursive definition.

$$RSD(R, X) = \begin{cases} \sum_{x \in X} \frac{1}{|X|} x & \text{if } R = \emptyset, \\ \sum_{i=1}^n \frac{1}{|R|} RSD(R_{-i}, \max(X)) & \text{otherwise,} \end{cases}$$

and $RSD(R) = RSD(R, A)$. The formal definition of *RD* is a special case of the above definition of *RSD*. In contrast to deterministic dictatorships, *RSD* is anonymous and is frequently used in subdomains of social choice that are concerned with the fair assignment of objects to agents (see, e.g., Budish et al., 2013).

3. Efficiency and strategyproofness

In order to reason about the outcomes of SDSs, we need to make assumptions on how agents compare lotteries. A common way to extend preferences over alternatives to preferences over lotteries is *stochastic dominance (SD)*. A lottery SD -dominates another lottery if, for every alternative x , the former is at least as likely to yield an alternative at least as good as x as the latter. Formally,

$$p \succsim_i^{SD} q \text{ iff for all } x \in A, \sum_{y: y \succsim_i x} p(y) \geq \sum_{y: y \succsim_i x} q(y).$$

It is well-known that $p \succsim_i^{SD} q$ if and only if the expected utility for p is at least as large as that for q for every von Neumann–Morgenstern utility function consistent with $\succsim_i$.

Thus, for the preference relation $\succsim_i: a, b, c$, we for example have that

$$(2/3a + 1/3c) \succsim_i^{SD} (1/3a + 1/3b + 1/3c),$$

while $2/3 a + 1/3 c$ and b are incomparable.

In this section, we define the notions of efficiency and strategyproofness considered in this paper. The two notions of efficiency defined below are generalizations of Pareto-optimality in non-probabilistic social choice. An alternative is *Pareto-dominated* if there exists another alternative such that all agents weakly prefer the latter to the former with a strict preference for at least one agent. An SDS is *ex post efficient* if it assigns probability zero to all Pareto-dominated alternatives (see e.g., Gibbard, 1977; Bogomolnaia et al., 2005).

Second, we define efficiency with respect to stochastic dominance. A lottery p is *SD-efficient* if there is no other lottery q that is weakly *SD*-preferred by all agents with a strict preference for at least one agent, i.e., $q \succsim_i^{SD} p$ for all $i \in N$ and $q \succ_i^{SD} p$ for some $i \in N$. It is well-known that *SD*-efficiency is stronger than *ex post* efficiency. An SDS is *SD-efficient* if it returns an *SD*-efficient lottery for every preference profile (see, e.g., Bogomolnaia and Moulin, 2001; Aziz et al., 2014, 2015).

For better illustration, consider $A = \{a, b, c, d\}$ and the preference profile $R = (\succsim_1, \dots, \succsim_4)$, with

$$\begin{aligned} \succsim_1: \{a, c\}, b, d & \quad \succsim_2: \{b, d\}, a, c \\ \succsim_3: a, d, b, c & \quad \succsim_4: b, c, a, d. \end{aligned}$$

Observe that no alternative is Pareto-dominated, i.e., for instance the uniform lottery $1/4 a + 1/4 b + 1/4 c + 1/4 d$ is *ex post* efficient. On

¹ The allocation instance for which *RSD* violates *SD*-efficiency by Bogomolnaia and Moulin uses 4 agents and 4 objects. An allocation problem can be associated with a social choice problem by letting the set of alternatives be the set of deterministic allocations and postulating that agents are indifferent among all allocations in which they receive the same object. Using this construction, the example by Bogomolnaia and Moulin translates to a social choice instance for which *RSD* fails *SD*-efficiency with 4 agents and $4! = 24$ alternatives. Aziz et al. (2013b) provide a similar example with 4 agents and 4 alternatives which is minimal in both parameters. For further discussion on the connection between the assignment setting and the social choice setting, we refer to Aziz et al. (2013a).

² For example, this also explains why another strategyproof extension of *RD* to weak preferences, the maximal recursive rule (Aziz, 2013), violates efficiency.

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