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Efficiency of Lowest-Unmatched Price Auctions

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HIGHLIGHTS

- In a LUPA, the winning bid is the lowest one among those submitted by only one player.
- All bidders pay a fee; the auctioneer retains the item if there is no winner.
- Despite apparent unfairness, expected payoffs of bidders and organizers are zero.
- LUPAs act as a price revealing mechanism in expected terms.

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ABSTRACT

In Lowest-Unmatched Price Auctions (LUPA) all participants pay a bidding fee and the lowest bid placed by only one participant wins. Many LUPAs do not specify what happens with the item on offer if there is no unmatched bid. The item may remain with the auctioneer which may appear unfair given that the auctioneer collects the bidding fees. We show that in a symmetric Nash equilibrium of a LUPA with known prize both players and the auctioneer will have an expected profit of zero. Moreover, LUPAs may be seen as a value-revealing mechanism.

1. Introduction

A Lowest-Unmatched Price Auction (LUPA) specifies that the winning bid is the lowest among all unmatched bids, i.e. those placed by only one player. Although participating in a LUPA requires strategic skills, they may be mistaken for gambles (see Raviv and Virag, 2009 and the clarifications by the Gambling Commission in 2008). Efficiency of LUPAs as trading mechanisms has not been investigated, except for a partial characterization by Scarsini et al. (2010) who employ the zero-sum property to derive that the organizer's expected payoff is non-positive (non-negative for participants); yet, according to them, LUPAs seem to generate more money than the value of the object auctioned. Besides, in most LUPAs bidders pay participation fees and with a strictly positive probability there is no winner, in which case the organizer would retain the item.[1] This rule seems unfair, as the organizer also obtains the bidding fees. In this paper we explicitly show that the expected payoff of participants and organizers is exactly zero.

From a strategic perspective, LUPAs give bidders incentives to outguess bids of their rivals, unlike, say, first (or second) price private value auctions, where bidders have incentives to reveal their valuation of the auctioned item through bids placed. We show that despite this, the value of the auctioned item is reflected in the bidding behavior, and conclusions can be drawn about the bidders' valuation of the item, as well as about efficiency of LUPAs as trading mechanisms.

In some LUPAs, organizers specify a total number of bids that need to be placed in order for the sale to take place. This number is typically high enough to cover the cost of the auctioned item

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[1] This rule was used in Eichberger and Vinogradov (2008, 2015) and Östling et al. (2011).

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Table 1
Summary of some LUPAs run in Germany in 2005–2006.

Media	Item	Value, €	Total bids	Winner, €	Profit, €
Radio	monetary	10 000	47 872	14.55	13 457.28
Radio	monetary	10 000	52 847	14.65	15 895.03
Radio	monetary	1 000	1 798	0.60	−118.98
Radio	monetary	3 000	6 732	5.82	298.68
Radio	monetary	5 000	6 201	11.16	−1961.51
Newspaper	bike	1 099	1 272	1.51	−475.72
TV	car	20 000	266 824	20.65	11 074.80
Radio	house	350 000	610 104	99.82	−51 049

through bidding fees. In others, the number of required bids is not specified. In Table 1, 4 out of 8 LUPAs of the second type resulted in losses for the organizers, while the remaining auctions were profitable.[2] Games with high and relatively low stakes seem equally likely to be profitable or unprofitable, independent of the media (newspaper, radio or TV) used as the auction's platform.

Papers dealing with LUPAs often assume that players are only allowed to place a single bid, see Rapoport et al. (2007), Östling et al. (2011) and Houba et al. (2011). In contrast, here, as in Eichberger and Vinogradov (2008, 2015) and Scarsini et al. (2010), the game has no restrictions on the number of bids placed by each player.

2. The model

One can classify the LUPAs in Table 1 into three groups. Firstly, these are LUPAs with a monetary prize. In these auctions the value of the prize is identical for all bidders and for the organizer, justifying the common value assumption. Secondly, there are LUPAs which sell standard items with a well-defined market value, as the bicycle in Table 1; web-based LUPAs would sell iPhones, iPads, cameras, or camcorders. In this case, a common value assumption can also be justified for the bidders; for the organizers the valuation may be different. If the organizer manufactures the item or obtains a bulk purchase discount from the manufacturer, he may procure the item at a price below its market value. Thirdly, some LUPAs sell items that different participants are likely to value differently, e.g., a tuned car or a house.

We begin our analysis with the common value case, proceed with the case of a different valuation by the organizer, and finally apply results to the private value variant of a LUPA. The formulation of the game follows closely Eichberger and Vinogradov (2015), henceforth EV (2015).

2.1. The game

Consider a finite set of identical potential bidders $I = \{1, \ldots, N\}$ who value the item which is to be sold in the auction at A. A player faces bidding costs of c per bid. Players may become active bidders or may choose not to bid at all.

We allow for multiple bids and define a strategy s_i of player i by a vector of binaries $s_i = (1, 0, \ldots, 0, 1, \ldots)$. The position b in strategy s_i refers to the bid b and $s_i(b)$ indicates whether player i places bid b $(s_i(b) = 1)$ or does not place this bid $(s_i(b) = 0)$. Bidding above A is unprofitable, therefore bids above A are dominated by the non-participation option, hence the number of undominated bids is finite. We denote the highest undominated bid by $\bar{b}$. Formally, a strategy is a mapping $s_i : \mathbb{N}(\bar{b}) \to \{0, 1\}$, where $\mathbb{N}(\bar{b})$ denotes the set of integers up to $\bar{b}$. The strategy set S of each player is the set of these mappings.

A strategy combination $\mathbf{s} = (s_1, s_2, \ldots, s_N)$ is an element of S^N. A strategy combination $\mathbf{s}$ can be written $(s_i, \mathbf{s}_{-i})$, where $\mathbf{s}_{-i}$ is the strategy combination played by player i's rivals. Given a strategy combination $\mathbf{s}$, one can determine the lowest unmatched bid $\mu(\mathbf{s})$. If there is no unmatched bid, we assign $\mu(\mathbf{s}) = 0$.

Let $\pi_i \in \Delta(S)$ be a mixed strategy of player i and denote by π_{-i} the combination of mixed strategies of his rivals. For a pure strategy s_i of player i a profile π_{-i} of mixed strategies of his rivals determines the probability $w_b(s_i, \pi_{-i})$ of bid b winning: $w_b(s_i, \pi_{-i}) = \sum_{\{\mathbf{s}_{-i}:\mu(s_i,\mathbf{s}_{-i})=b\}} \pi(\mathbf{s}_{-i})$, where $\pi(\mathbf{s}_{-i}) = \prod_{i\neq j} \pi(s_j)$ denotes the probability of the respective strategy combination. With this notation, the expected payoff of player i from playing s_i is

$$P_i(s_i, \pi_{-i}) = \sum_{b=1}^{\bar{b}} s_i(b)\left[(A-b)\, w_b(s_i, \pi_{-i}) - c\right]. \tag{1}$$

Defining the expected payoff $P_i(\pi_i, \pi_{-i})$ of a mixed strategy π_i for player i in the usual way, one can apply the standard definition of a Nash equilibrium in mixed strategies. A combination of mixed strategies $\left(\pi_i^*, \pi_{-i}^*\right)$ is a Nash equilibrium if

$$P_i\left(\pi_i^*, \pi_{-i}^*\right) \geq P_i\left(\pi_i, \pi_{-i}^*\right) \quad \text{for all } \pi_i \in \Delta(S) \text{ and all } i \in I.$$

We will focus on symmetric equilibria and, therefore, omit the indices of players.[3] We are now ready to state our main results.

2.2. Bidders' equilibrium payoffs

In a symmetric Nash equilibrium π, all pure strategies s in the support of the equilibrium mixed strategy $\operatorname{supp}\pi$ must yield an equal expected payoff:

$$P(s, \pi) = P(s', \pi), \quad \text{for all } s, s' \in \operatorname{supp}\pi. \tag{2}$$

The following proposition establishes that, in equilibrium, players in a LUPA will face an expected payoff of zero. The proof rests on showing that the strategy s^0 of not bidding and, hence, obtaining a payoff of zero has positive probability in any symmetric Nash equilibrium. Hence, s^0 belongs to the support of the equilibrium mixed strategy and the expected payoff of all strategies which are played with positive probability in equilibrium must be zero.

Proposition 1. *For an equilibrium mixed strategy π, the expected payoff $P(s, \pi)$ is zero for all $s \in \operatorname{supp}\pi$.*

Proposition 1 establishes that participating in a LUPA is not profitable in expected terms. For an item auctioned in a LUPA which the bidders value at the market price A, risk-neutral players are indifferent between obtaining the good in the market and participating in the LUPA. In this sense, LUPA is a "*fair lottery*". This interpretation of LUPAs as trading mechanisms rests upon the option not to participate and instead to obtain the item in the market.

[2] Examples are from Eichberger and Vinogradov (2015). In all cases, the bidding cost is 49c per bid. Profit is calculated as the total revenue from bids less the (advertized) value of the prize. On top of this, there was a fee paid by the organizers of the auction to service providers, typically 12c per bid.

[3] Formal definitions of $\mu(\mathbf{s})$ and $w_b(s_i, \pi)$ and a more detailed discussion are in EV (2015).