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A Stein-like estimator for linear panel data models

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HIGHLIGHTS

- A Stein-like estimator for linear panel data models is proposed in this paper, asymptotics for this Stein-like estimator is also established.
- We show the asymptotic risk of the Stein-like estimator is strictly smaller than the fixed effects estimator within a local asymptotic framework.
- Monte Carlo simulations confirm the theoretical findings in the paper.

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ABSTRACT

In this paper we follow Hansen (2015a) and propose a Stein-like estimator for linear panel data models. Our estimator takes a weighted average of the fixed effects estimator and the random effects estimator using the weights constructed from Hausman's (1978) testing statistic. We establish the asymptotic distribution of the Stein-like estimator and show its asymptotic risk being strictly smaller than the fixed effects estimator within a local asymptotic framework.

1. Introduction

Since Hausman (1978) has proposed the standard Hausman test, a well-known and widely used method to make the choice between the random effects (RE) and fixed effects (FE) estimator, many variants have been proposed for linear panel data models. For example, see Hahn et al. (2011), and Frondel and Vance (2010) among others. Recently several econometricians examine the Hausman pretest and suggest some pretest estimators. Guggenberger (2010) studies the impact of a Hausman pretest on the size of t-test in a panel data model and finds substantial size distortion due to the possible poor power properties of the pretest; Kabaila et al. (2015) assess the finite sample impact of Hausman pretest on the coverage probability of a confidence interval for the slope parameter and show that the pretest estimator leads to this confidence interval having minimum coverage probability far below nominal. In the framework of Hausman and Taylor's (1981) model where only some of the variables may be correlated with the individual effects, Baltagi et al. (2003) suggest an alternative pretest estimator and investigate its finite sample performance using Monte Carlo experiments. In the case of instrumental variable regression, Chmelarova and Hill (2010) show that a small probability of Type I error may reduce MSE of pretest estimator when a Hausman pretest is used to choose the least squares or instrumental variable estimators by a Monte Carlo experiment. However, it is still not very clear about the properties of these pretest estimators.

Instead of making a decision to use one or another estimator through a test, some estimates based on their linear combinations have gained interests in the statistical and econometric literature. It has been shown that these estimators can reduce the risks and have superiority. Following Stein (1956) and James and

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Stein (1961), various modified Stein-like estimators have been developed in the literature. Massoumi (1978) develops a Stein-like estimator for the reduced form coefficients of simultaneous equations which combines the corresponding restricted 3SLS and the unrestricted LS estimators. It shows that the modified Stein-like estimator has finite moments and thus bounded risk. More recently, Hansen (2015a) extends Massoumi's idea to the single equation instrumental variables models, and proposes a shrinkage estimator which combines OLS and 2SLS estimators, and finds that the asymptotic risk of the estimator is strictly less than the one of the 2SLS estimator when the number of endogenous variables exceeds two.

The aim of our paper is to develop an alternative Stein-like estimator for linear panel data models and study the risk of various estimators. Our estimator takes a weighted average of the FE estimator and RE estimator using the weights constructed from Hausman's testing statistic. We establish its asymptotic distribution and show that the asymptotic risk of this Stein-like estimator is strictly smaller than the FE estimator under some conditions of shrinkage parameter.

The remainder of the article is as follows. Section 2 presents the model and the Stein-like estimator. In Section 3, the asymptotic distributions of the estimators are summarized, and the asymptotic risks of various estimators are studied within a local asymptotic framework. Section 4 presents a simulation study. Section 5 concludes. Proofs of the theoretical results are contained in the Appendix.

Notation: Let I_n be the $n \times n$ identity matrix and ι_n be the $n \times 1$ vector of ones for any positive integer n. Let 0_K and $0_{K\times K}$ be the $K \times 1$ vector of zeros and the $K \times K$ matrix of zeros, respectively. $\|a\| = \left(a'a\right)^{1/2}$ denotes the Euclidean norm for a $K \times 1$ vector a.

2. Stein-like estimator for panel data models

In this paper we consider the standard panel data model with one-way error component

$$y_{it} = \alpha + X_{it}'\beta + u_{it}, \tag{2.1}$$

$$u_{it} = \mu_i + v_{it}, \tag{2.2}$$

$i = 1, \ldots, N,\ t = 1, \ldots, T$, where i and t denote the cross-section dimension and time series dimension, respectively, X_{it} is a $K \times 1$ vector of time-varying explanatory variables, the unobservable individual effect μ_i is time-invariant and distributed independently across i with zero mean and variance σ_μ^2, the disturbance v_{it} is assumed uncorrelated with $\{(X_{js}, \mu_j) : j = 1, \ldots, N,\ s = 1, \ldots, T\}$ and has zero mean and variance σ_v^2; α is a scalar constant, and β is a $K \times 1$ vector of unknown parameters. We are interested in the estimation of β when N goes to ∞ and T is small.

Let $Z_{it} = \left(1, X_{it}'\right)'$. Denote matrices (or vectors) y_i, X_i, Z_i, u_i and v_i with T rows and their t-th rows are given by $y_{it}, X_{it}', Z_{it}', u_{it}$, and v_{it}, respectively. In vector form, the model can be written as

$$y = \alpha\iota_{NT} + X\beta + u = Z\delta + u,$$
$$u = Z_\mu\mu + v,$$

where $u = \left(u_1', \ldots, u_N'\right)'$, y, X and v are defined similarly, $Z = [\iota_{NT}, X]$, $Z_\mu = I_N \otimes \iota_T$, $\delta = (\alpha, \beta')'$ and $\mu = (\mu_1, \ldots, \mu_N)$. Let $P_{Z_\mu} = Z_\mu\left(Z_\mu'Z_\mu\right)^{-1}Z_\mu'$ and $Q_{Z_\mu} = I_{NT} - P_{Z_\mu}$.

Now we state the most commonly used estimators for the linear panel data models in (2.1)–(2.2). The *random effects estimator* of β is given by

$$\widehat{\beta}_{RE} = \left(X'RX\right)^{-1}X'Ry \tag{2.3}$$

where $R = Q_{Z_\mu} + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_v^2 + T\hat{\sigma}_\mu^2}\left(P_{Z_\mu} - P_{\iota_{NT}}\right)$, $\hat{\sigma}_v^2$ and $\hat{\sigma}_\mu^2$ are estimators for σ_v^2 and σ_μ^2, respectively. Alternatively, the *fixed effects estimator* is defined as

$$\widehat{\beta}_{FE} = \left(X'Q_{Z_\mu}X\right)^{-1}X'Q_{Z_\mu}y. \tag{2.4}$$

Given the RE estimator (2.3) and FE estimator (2.4), Hausman's testing statistic is given by

$$H_N = N\left(\widehat{\beta}_{FE} - \widehat{\beta}_{RE}\right)'\left(\widehat{V}_{FE} - \widehat{V}_{RE}\right)^{-1}\left(\widehat{\beta}_{FE} - \widehat{\beta}_{RE}\right),$$

where $\widehat{V}_{FE} = \left(X'Q_{Z_\mu}X\right)^{-1}\tilde{\sigma}_v^2$ and $\widehat{V}_{RE} = \left(X'RX\right)^{-1}\hat{\sigma}_v^2$ are estimators of the asymptotic variance terms for the FE estimator and RE estimator, respectively, and $\tilde{\sigma}_v^2$ may be an estimator of σ_v^2 and different from $\hat{\sigma}_v^2$. Based on Hausman's testing statistic, the pretest estimator is

$$\widehat{\beta}_{Pretest} = w_p\widehat{\beta}_{RE} + \left(1 - w_p\right)\widehat{\beta}_{FE}$$

where $w_p = 1(H_N \le \chi^2_{K,1-\alpha_p})$, $1(\cdot)$ is the usual indicator function, α_p is the nominal level of chi-square distribution with K degrees of freedom (χ_K^2). When $\overline{X}_i$ and μ_i are uncorrelated, it can be shown that $\widehat{\beta}_{FE}$ and $\widehat{\beta}_{RE}$ are both consistent estimators of β and H_N follows asymptotically χ_K^2.

Now, we apply the same idea as Hansen (2015a) to the panel case and propose a *Stein-like estimator* for β as follows

$$\widehat{\beta}_{Stein} = w_s\widehat{\beta}_{RE} + (1 - w_s)\widehat{\beta}_{FE}, \tag{2.5}$$

where $w_s = \frac{\tau}{H_N}1(H_N \ge \tau) + 1(H_N < \tau)$, τ is a positive shrinkage parameter. We can see that when $K > 2$, $\tau = K - 2$ is recommended according to Theorem 3.2 (iv) in the next section. Clearly, the above estimator can be written as a *positive-part James–Stein* estimator:

$$\widehat{\beta}_{Stein} = \widehat{\beta}_{RE} + \left(1 - \frac{\tau}{H_N}\right)_+\left(\widehat{\beta}_{FE} - \widehat{\beta}_{RE}\right),$$

where $(a)_+ = a \cdot 1(a > 0)$. Similarly, the pretest estimator can be expressed as

$$\widehat{\beta}_{Pretest} = \widehat{\beta}_{RE} + 1\left(\frac{\chi^2_{K,1-\alpha_p}}{H_N} < 1\right)\left(\widehat{\beta}_{FE} - \widehat{\beta}_{RE}\right).$$

These two estimators can be seen as shrinkage estimators because they both shrinkage an unrestricted estimator $\widehat{\beta}_{FE}$ to a restricted estimator $\widehat{\beta}_{RE}$ with different weights. When H_N is small enough, say $H_N \le \min\left(\chi^2_{K,1-\alpha_p}, \tau\right)$, Stein-like estimator and pretest estimator give the same weight 1 to RE estimator; when H_N is large enough, say $H_N > \max\left(\chi^2_{K,1-\alpha_p}, \tau\right)$, Stein-like estimator puts smaller weight on RE estimator than the pretest estimator.

3. Asymptotic properties for panel Stein-like estimator

To establish the asymptotic distribution of $\widehat{\beta}_{Stein}$, we follow Hansen (2015a,b) and develop the theorems under the local asymptotic framework. Assume

$$\mathrm{Cov}\left(\overline{X}_i, \mu_i\right) = N^{-1/2}r, \tag{3.1}$$

where $\overline{X}_i = T^{-1}\sum_{t=1}^{T}X_{it}$ and r is a $K \times 1$ vector of constants. Clearly, μ_i's are random effects when $r = 0$ and (locally) fixed effects when $r \neq 0$.