Economics Letters 141 (2016) 162–165

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Optimal auctions with endogenous budgets

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HIGHLIGHTS

- We study standard IPV auctions when bidders endogenously determine their budgets.
- Prior to bidding, bidders decide how much money to borrow. Borrowing is costly.
- We obtain a revenue ranking of standard auctions: all-pay>first-price> second-price.
- The optimal auction is an all-pay auction with suitably chosen reserve price.

ARTICLE INFO

Article history:
Received 13 November 2015
Received in revised form
10 February 2016
Accepted 16 February 2016
Available online 2 March 2016

JEL classification:
D44
D47
D82

Keywords:
Optimal auction
All-pay auction
Budget constraints
Liquidity

ABSTRACT

We study the benchmark independent private value auction setting when bidders have endogenously determined budgets. Before bidding, a bidder decides how much money she will borrow. Bidders incur a cost to borrowing. We show that bidders are indifferent between participating in a first-price, second-price and all-pay auction. The all-pay auction gives higher revenue than the first-price auction, which gives higher revenue than the second price auction. In addition, when the distribution of values satisfies the monotone hazard rate condition, the revenue maximizing auction is implemented by an all-pay auction with a suitably chosen reserve price.

1. Introduction

The seminal papers on auction design assume that a bidder's ability to pay for a good exceeds her willingness to pay; preferences are quasilinear.[1] Yet, in many well-studied auction markets, this restriction does not hold—bidders face binding budget constraints. Authors have argued that the presence of budgets limits the applicability of auction theory in real-world settings.[2] In response, a literature developed that analyzed how the presence of budgets changes the auction design problem. For example, Che and Gale (1996, 1998) compare standard auctions with budgets, and Laffont and Robert (1996), Che and Gale (2000), and Pai and Vohra (2014) construct revenue-maximizing auctions when bidders have budgets.[3]

The above literature assumes that budgets are exogenously determined.[4] In practice, however, bidders can choose the amount of resources devoted towards bidding in the auction. Bidding in the auction requires liquid resources, which can be obtained by borrowing or diverting resources away from alternative profitable investments. Thus, even if a bidder does not borrow from a bank, she still incurs an opportunity cost of funds. This raises the question: what is the optimal selling mechanism when buyers

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1 See Myerson (1981).

2 See Rothkopf (2007).

3 This paper studies the case of private values, as do the previously cited works. There is additional work that studies bidding in auctions with budgets in interdependent value environments. For example, Fang and Parreiras (2003) and Kotowski and Li (2014).

4 Two exceptions are Burkett (forthcoming-a,b), where budgets are endogenously determined in a principal–agent relationship, and Rhodes-Kropf and Viswanathan (2005), who consider a variety of forms of endogenous financing in a first-price auction for a risky asset.

http://dx.doi.org/10.1016/j.econlet.2016.02.017

endogenously make such liquidity choices? This question is also important not only for the auctions literature, but also for the growing literature in monetary economics that models liquidity choices for decentralized trade.[5]

In this paper, we study auction design with endogenously determined budgets. We consider an auction for an indivisible good, where bidders have independent private values, and endogenously determine their budgets after observing their private information. Borrowing is costly, and a bidder incurs a cost of borrowing whether or not her bid wins.

We show that bidders are indifferent between competing in the first-price, second-price and all-pay auctions. However, the auctions are not revenue-equivalent. The all-pay auction has the highest expected revenue, and the second-price auction has the lowest expected revenue. This is the same revenue ranking that we see in the exogenous budget case of Che and Gale (1996), but the intuition is distinct. In Che and Gale (1996), the all-pay auction yields higher revenues than first or second price because the budget constraint is less likely to bind in the all-pay. In our model, the all-pay auction yields higher revenues because it economizes on bidders' borrowing costs.

On the design of revenue-maximizing auctions, we show that the optimal auction is implemented by an all-pay auction with a suitably chosen reserve price.[6] This differs from an optimal auction with exogenous budgets: the latter would not necessarily sell the good to the highest-value bidder.[7] The reason for the difference is that, with exogenous budgets, high-value bidders are unable to express high demand for the good. With endogenous budgets, high-value bidders are able to reveal that they have a higher demand by borrowing more money to place higher bids. While placing higher bids comes at a cost, the auctioneer can minimize these costs by using an all-pay payment scheme.

2. Model

2.1. Environment

A seller has one unit of an indivisible good, which she has no value for. There are $N \geq 2$ risk-neutral potential bidders. A bidder's preferences are described her valuation $v \in \mathbb{R}_+$. Bidder valuations are *i.i.d.* draw of a random variable with density f. We assume that f has full support over $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$. Thus, f has an associated distribution function F which is continuous and strictly increasing over $[\underline{v}, \overline{v}]$, with $F(\underline{v}) = 0$ and $F(\overline{v}) = 1$.

A bidder determines her budget after finding out her value, but before placing her bid and observing competing bids. If a bidder borrows b, she must repay $b + c(b)$, where $c(b)$ is continuous, differentiable, strictly increasing, and weakly convex.[8]

Therefore, if bidder i wins the good, pays p and borrows b, where $b \geq p$, her utility is

$$v_i - p - c(b).$$

A bidder cannot place a bid that exceeds the amount of money that she has borrowed.

2.2. Mechanisms

By the revelation principle, we limit attention to direct revelation mechanisms. Given the profile of reported types $\mathbf{v} = (v_1, \ldots, v_N)$, the direct revelation mechanism states a bidder's (ex-post) probability of winning $Q_i(\mathbf{v})$, (ex-post) expected transfer $T_i(\mathbf{v})$, and borrowing $b_i(v_i)$. The amount a bidder borrows is independent of other bidders' reported types, because a bidder decides her budget before bidding. Since borrowing is costly, bidders do not borrow more than they would need to pay in the auction; hence $b_i(v) = \sup_{v_{-i}} T_i(\mathbf{v})$. Feasibility requires

$$\sum_{i=1}^{N} Q_i(\mathbf{v}) \leq 1.$$

Let $q_i(v) = \mathbb{E}_{v_{-i}}(Q_i(v, v_{-i}))$ be bidder i's interim probability of winning when reporting type v. Similarly, $t_i(v) = \mathbb{E}_{v_{-i}}(T_i(v, v_{-i}))$ denotes the interim expected payment made by bidder i. Therefore, the expected utility of bidder i, if her true type is v_i and she reports type v, is

$$U_i(v, v_i) = q_i(v)v_i - t_i(v) - c(b_i(v)).$$

The direct revelation mechanism is (interim) incentive-compatible if

$$U_i(v_i, v_i) \geq U_i(v, v_i) \quad \forall v \in [\underline{v}, \overline{v}],\ i = 1, \ldots, N.$$

3. Standard auctions

Incentive compatibility implies that $q_i(v)$ and $t_i(v)$ are weakly increasing. Thus, both functions are differentiable almost everywhere along $[\underline{v}, \overline{v}]$. Pick any point where $U_i(v, v_i)$ is differentiable with respect to v at v_i. The necessary first order condition for incentive compatibility implies

$$\frac{\partial U_i(v, v_i)}{\partial v} = q_i'(v)v_i - t_i'(v) - c'(b_i(v))b_i'(v) = 0.$$

Therefore, the total derivative of $U_i(v_i, v_i)$ with respect to v_i is

$$\frac{dU_i(v_i, v_i)}{dv_i} = q_i(v_i).$$

Since U_i is differentiable almost everywhere, this implies

$$U_i(v_i, v_i) = U_i(\underline{v}, \underline{v}) + \int_{\underline{v}}^{v_i} q_i(s)ds. \tag{1}$$

Thus, we can use standard Myersonian approach to characterize bidder i's interim expected utility. Eq. (1) implies that bidder i is indifferent between any two mechanisms that give the same interim probability of winning, and give the same expected utility to the low type. In particular, bidders are indifferent between participating in the first-price, second-price, and all-pay auction.

We use Eq. (1) to establish a revenue ranking of the three standard auctions. We consider symmetric Bayes Nash Equilibria.

Proposition 1 (*Revenue Ranking of Standard Auctions*)***.*** *The all-pay auction has strictly greater expected revenue than the first price auction. The first price auction has strictly greater expected revenue than the second price auction.*

Proof. Since bids are strictly increasing in each auction, $q(v) = F(v)^{N-1}$, and $U(\underline{v}, \underline{v}) = 0$. Let $\beta^f(v_i)$, $\beta^s(v_i)$, and $\beta^a(v_i)$ be the symmetric equilibrium bid functions in the first price auction, the second price auction, and the all-pay auction, respectively.

In each auction, a bidder never pays in excess of her bid. Thus, a bidder borrows exactly the amount she bids: $\beta^j(v_i) = b^j(v_i)$ for $j = f, s, a$.

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5 Our paper's environment, in which agents choose their money holdings before engaging in trade, is reminiscent of the models in Lagos and Wright (2005) and Rocheteau and Wright (2005). Lagos et al. (forthcoming) provide a recent survey. In this literature, the pricing mechanism is crucial for determining real balances, output, and efficiency. Galenianos and Kircher (2008), also introduced auctions into monetary models. However, this literature assumes that goods are sold by second-price auctions, without examining whether it is optimal.

6 This requires that bidders have monotone virtual values, as in Myerson (1981).

7 For example, in Laffont and Robert's (1996) optimal auction, all bidders with values above some threshold win with equal probability.

8 When determining the optimal auction, we assume that c is linear.