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A nonparametric unit root test under nonstationary volatility

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HIGHLIGHTS

- A nonparametric unit root test robust to nonstationary volatility is proposed.
- The proposed test statistic does not require a correction of serial correlation.
- The proposed test is correctly sized and has desirable power.
- In finite sample properties, our test outperforms other tests in the literature.

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ABSTRACT

We develop a new nonparametric unit root testing method that is robust to permanent shifts in innovation variance. Unlike other methods in the literature, our test does not require a parametric specification or lag/bandwidth selection to adjust for serial correlation.

1. Introduction

Recent body of empirical evidence indicates that variance shifts (nonstationary volatility) is a common occurrence in macroeconomic and financial data; see Busetti and Taylor (2003), McConnell and Perez-Quiros (1998) and Sensier and Van Dijk (2004). This finding coupled with nonstationarity in the levels of these types of data led the researchers to investigate the impact of variance shifts on unit root tests. In one of these studies, Cavaliere and Taylor (2007), henceforth CT, document that under nonstationary volatility, the asymptotic distributions of standard unit root tests are altered by the inclusion of a new nuisance parameter called the "variance profile", leading to size distortions in these tests. In order to achieve correct inference, CT suggest first consistently estimating this nuisance parameter and then updating the asymptotic distribution of Phillips and Perron's (1988) tests with this estimate. While their inclusion of the new nuisance parameter generates significant gains in size over classical unit root tests, they still rely on the methodologies used in earlier studies to correct for other nuisance parameters such as serial correlation in errors. CT adjust their test statistic via the estimation of the long run variance, obtained by a semi-parametric kernel or a parametric ADF based regression estimation. The success of these methods highly depends on lag length, bandwidth and Kernel selection in terms of finite sample properties. In this paper, we propose a nonparametric unit root test that is robust to nonstationary volatility problem yet does not require a long run variance estimation.

We derive our test statistic by modifying Nielsen's (2009) nonparametric variance ratio statistic with the nonparametric variance profile estimator of CT. Computation of the proposed test statistic involves a fractional transformation of observed series, but it does not require any parametric regression or the choice of any tuning parameters like lag length and bandwidth. Therefore, we not only modify Nielsen's test to be robust against nonstationary volatility, but also improve on the finite sample properties of CT

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statistic for all considered types of serial correlation. Derivation of the limiting distribution of fractionally integrated processes with nonstationary volatility and the proofs are placed in Appendix.[1]

2. Model and variance ratio test

2.1. Model

Let $\{x_t\}_{t=0}^T$ be generated by:

$$x_t = y_t + \theta'\delta_t \tag{1}$$

$$y_t = \rho y_{t-1} + u_t \tag{2}$$

$$u_t = C(L)\varepsilon_t \tag{3}$$

$$\varepsilon_t = \sigma_t e_t \tag{4}$$

where $e_t \sim i.i.d.(0, 1)$ and $\theta'\delta_t$ is the deterministic term and $C(L)$ is the lag polynomial. From CT, we have following assumptions:

Assumption. $\mathcal{A}.1$ The lag polynomial $C(L) \neq 0$ for all $|L| \leq 1$, and $\sum_{j=0}^{\infty} j|c_j| < \infty$. $\mathbb{E}|e_t|^r < K < \infty$ for some $r \geq 4$.

$\mathcal{A}.2$ ρ satisfies $|\rho| \leq 1$.

$\mathcal{A}.3$ σ_t satisfies $\sigma_{\lfloor Ts \rfloor} := \omega(s)$ for all $s \in [0, 1]$, where $\omega(.) \in \mathcal{D}$ is non-stochastic and strictly positive. For $t < 0$, σ_t is uniformly bounded, that is there exists a σ^* such that $\sigma_t \leq \sigma^* < \infty$.

The assumptions $\mathcal{A}.1$ and $\mathcal{A}.2$ are very standard in unit root testing literature. CT characterize the dynamics of innovation variance in $\mathcal{A}.3$, which should be bounded and display a countable number of jumps.

A fundamental object that is defined in CT is given below:

$$\eta(s) := \left(\int_0^1 \omega(r)^2 dr\right)^{-1} \left(\int_0^s \omega(r)^2 dr\right). \tag{5}$$

This object is referred to as the *variance profile* of the process. Further, CT show that $\int_0^1 \omega(r)^2 dr = \bar{\omega}^2$ is the limit of $T^{-1}\sum_{t=1}^T \sigma_t^2$.

2.2. Variance Ratio test under nonstationary volatility

So as to modify the Variance Ratio test (Nielsen, 2009) statistic we first need the fractional partial sum operator for some $d > 0$:

$$\tilde{x}_t := \Delta_+^{-d} x_t = (1-L)_+^{-d} x_t = \sum_{k=0}^{t-1} \frac{\Gamma(k+d)}{\Gamma(d)\Gamma(k+1)} x_{t-k}$$

$$= \sum_{k=0}^{t-k} \pi_k(d) x_{t-k} \tag{6}$$

where $\Gamma(.)$ is gamma function. Under the assumptions $\mathcal{A}$, following lemmas hold:

Lemma 1. *Assume that $\{u_t\}_{t=0}^T$ is generated by (3)–(4) and $\rho = 1 - c/T$ with $c \geq 0$.*

i. *$y_T(t) = T^{-1/2}\sum_{k=1}^{\lfloor Tt \rfloor} e^{-c(\lfloor Tt \rfloor - k)} u_k \xrightarrow{w} \bar{\omega}C(1)J_\omega^c(t)$, where $J_\omega^c(t) = \int_0^t \exp(-c(t-s))dB_\omega(s)$ and $B_\omega(s) = \bar{\omega}^{-1}\int_0^s \omega(r)dB(r)$.*
ii. *$B_\omega(s) = B_\eta(s) := B(\eta(s))$ where $B_\eta(s)$ variance transformed Brownian motion, $\eta(s)$ is defined in (5). Thus, $J_\eta^c(t) := J_\omega^c(t) = \int_0^t \exp(-c(t-s))dB_\eta(s)$.*
iii. *For all $d > 0$, $\tilde{y}_T(t) = T^{-d}\Delta_+^{-d} y_T(t) \xrightarrow{w} \bar{\omega}C(1)J_{\omega,d}^c(t)$, where $J_{\omega,d}^c(t) = \Gamma(d+1)^{-1}\int_0^t (t-s)^d dJ_\omega^c(s)$. Further, we have $J_{\omega,d}^c(t) = J_{\eta,d}^c(t)$.*

[1] The notation in the paper follows Cavaliere and Taylor (2007).

Remark 1. Lemma 1(i) and (ii) are from Cavaliere (2005) and CT. Lemma 1(iii) is new and establishes weak convergence for fractionally integrated processes with non-stationary volatility. Although Demetrescu and Sibbertsen (2014) model the fractional integrated process with non-stationary volatility, they do not establish weak convergence of this object.

Remark 2. Note that under the null hypothesis of $\rho = 1$ or $c = 0$ the above variance transformed Uhlenbeck–Ornstein process becomes a variance transformed Brownian motion. For instance, under the null the partial sum process $\tilde{y}_T(t)$ will converge to $\bar{\omega}C(1)\int_0^t (t-s)^d dB_\eta(s)$ where we can define $B_{\eta,d}(t) := \int_0^t (t-s)^d dB_\eta(s)$. This limiting distribution resembles the type II fractional Brownian motions defined by Marinucci and Robinson (2000), since $B_{\eta,d}(t)$ does not contain any pre-historic influence (see also Wang et al., 2002).

Like Nielsen (2009), we apply OLS detrending to the observed series x_t to clean out the deterministic terms. Let $\hat{x}_t$ be the OLS detrended residuals and defining $\tilde{\hat{x}}_t = \Delta_+^{-d}\hat{x}_t$, our test statistic is then given by:

$$\tau_\eta(d) = T^{2d}\frac{\sum_{t=1}^T \hat{x}_t^2}{\sum_{t=1}^T \tilde{\hat{x}}_t^2}. \tag{7}$$

Theorem 1. *Assume that the time series $\{x_t\}$ is generated by Eqs. (1)–(4) and $\rho = 1 - c/T$ for $c \geq 0$. Let $j = 0$ when $\delta_t = 0$, $j = 1$ when $\delta_t = 1$ and when $\delta_t = [1, t]'$ for $d > 0$*

i. *$\hat{x}_T(t) \xrightarrow{w} J_{\eta,j}^c(t)$ where $J_{\eta,j}^c(t) = J_\eta^c(t) - \left(\int_0^1 J_\eta^c(s)D_j(s)'ds\right)\left(\int_0^1 D_j(s)D_j(s)'ds\right)^{-1} D_j(t)$ for $j = 1, 2$, and $D_1(s) = 1$, $D_2(s) = [1, s]'$ and $J_{\eta,0}^c(t) = J_\eta^c(t)$.*
ii. *$\tilde{\hat{x}}_T(t) \xrightarrow{w} J_{\eta,d,j}^c(t)$ where $J_{\eta,d,j}^c(t) = J_{\eta,d}^c(t) - \left(\int_0^1 J_{\eta,d}^c(s)D_j(s)'ds\right)\left(\int_0^1 D_j(s)D_j(s)'ds\right)^{-1}\int_0^t \frac{(t-r)^{d-1}}{\Gamma(d)}D_j(r)dr$ for $j = 1, 2$. Further $J_{\eta,d,0}^c(t) = J_{\eta,d}^c(t)$.*
iii. *$\tau_\eta(d) = T^{2d}\frac{\sum_{t=1}^T \hat{x}_t^2}{\sum_{t=1}^T \tilde{\hat{x}}_t^2} \xrightarrow{w} U_{j,\eta}(d) = \frac{(\bar{\omega}C(1))^2\int_0^1 J_{\eta,j}^c(s)^2 ds}{(\bar{\omega}C(1))^2\int_0^1 J_{\eta,d,j}^c(s)^2 ds} = \frac{\int_0^1 J_{\eta,j}^c(s)^2 ds}{\int_0^1 J_{\eta,d,j}^c(s)^2 ds}$.*

Remark 3. Note that short run dynamics cancel out in asymptotic distribution since the numerator and the denominator share the same long run variance component in part (iii).

2.3. Simulated asymptotic distribution

The test statistic obtained in Theorem 1 involves $\eta(s)$ as nuisance parameter which can be consistently estimated by modifying the nonparametric estimator in CT:

$$\hat{\eta}(s) := \frac{\sum_{t=1}^{\lfloor Ts \rfloor}(\Delta\hat{x}_t)^2 + (Ts - \lfloor Ts \rfloor)(\Delta\hat{x}_{\lfloor Ts \rfloor+1})^2}{\sum_{t=1}^T (\Delta\hat{x}_t)^2}. \tag{8}$$

Theorem 2. *Under the conditions of Theorem 1*

i. *(CT show) $B_{\hat{\eta},T}(s) := T^{-1/2}\sum_{t=1}^{\lfloor(\hat{\eta}\lfloor Ts \rfloor/T)T\rfloor} e_t \xrightarrow{w} B_\eta(s)$.*
ii. *$B_{\hat{\eta},d,T}(s) := T^{-d}\Delta_+^{-d}B_{\hat{\eta},T}(s) \xrightarrow{w} B_{\eta,d}(s)$.*