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Sieve bootstrap monitoring for change from short to long memory

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HIGHLIGHTS

- A monitoring procedure is proposed to sequentially detect changes from short to long memory processes.
- The proposed sieve bootstrap method is robust for many types of innovation processes.
- The new procedure exhibits competitive power than the existing retrospective test.

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ABSTRACT

This paper proposes a variance ratio statistic to monitor changes from short to long memory processes. The asymptotic distribution is derived under the null hypothesis and the consistency of the monitoring procedure is proven under the alternative hypothesis. A sieve bootstrap approximation method is introduced to determine the critical values. Simulations indicate that the new procedure is quite robust for many types of innovation processes and performs better than the existing retrospective test.

1. Introduction

During the past two decades, there is a growing body of evidence showing that economic and financial time series display changes in persistence. A number of testing procedures have been suggested that aim to distinguish such behavior. For surveys we refer the reader to Kim (2000), Leybourne et al. (2007), Halunga et al. (2009), Chen et al. (2012), among many others. All these tests stay in the classical $I(1)/I(0)$ framework. Sibbertsen and Kruse (2009), Kruse and Sibbertsen (2012), Martins and Rodrigues (2014), and Hassler and Meller (2014) have extended these results to the case of long memory for many economic variables exhibiting long-range dependencies that cannot be covered by the classical framework. Hassler and Nautz (2008) showed that the key policy rate of the European Central Bank has changed from a short memory to a long memory series. Hassler and Scheithauer (2011) applied a ratio test and a LBI test to detect the change point from short to long memory process. However, this work is based on retrospective test, that is, detecting change point in a fixed historical sample. As many economic and financial data arrive steadily and cheaply, the development of sequential tests is an important issue in change point analysis.

In this paper, we propose a variance ratio monitoring statistic to sequentially detect changes from short to long memory processes. The asymptotic distribution of the monitoring statistic is derived under the null hypothesis and the consistency of the procedure is proven under the alternative hypothesis. As the short memory process with many different types of innovation processes give the same asymptotic distribution, size distortions are not negligible in finite samples. To overcome this drawback, we propose a sieve bootstrap test procedure. Monte Carlo simulations confirm the validity of the proposed monitoring procedure.

2. Model and monitoring statistic

Let $y_1, y_2, \ldots,$ be an observed time series that can be decomposed as

$$y_t = \mu_t + x_t, \quad t = 1, 2, \ldots, \tag{1}$$

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where $\mu_t = E(y_t)$ is a deterministic component. For simplicity we restrict the analysis to the constant components, namely, $\mu_t = \mu$. An extension to polynomials in time would be possible. The term x_t is the random component. Similar to Hassler and Scheithauer (2011), we assume x_t satisfies usual invariance principles.

Assumption 1. Let x_t be an $I(0)$ process satisfying (as $T \to \infty$)

$$T^{-1/2}\sum_{t=1}^{[Ts]} x_t \Rightarrow B(s), \quad s \in [0,1],$$

$$T^{-1}\sum_{t=1}^{T} x_t^2 \xrightarrow{p} \sigma_0^2$$

where $B(\cdot)$ is a Brownian motion, and "$\Rightarrow$" and "$\xrightarrow{p}$" stand for weak convergence and convergence in probability, respectively.

Assumption 2. Let x_t be an $I(d)$ process satisfying (as $T \to \infty$)

$$T^{-1/2-d}\sum_{t=1}^{[Ts]} x_t \Rightarrow B_d(s), \quad s \in [0,1]$$

$$T^{-1}\sum_{t=1}^{T} x_t^2 \xrightarrow{p} \sigma_1^2$$

where $B_d(\cdot)$ is a type I fractional Brownian motion with long memory parameter $0 < d < 1.5$, $d \neq 0.5$.

We focus on the following change point problem: observe sequences $y_1, y_2, \dots,$ and detect whether a short to long memory change occurs in model (1), namely, we want to test the null hypothesis

$$H_0 : y_t \sim I(0), \quad t = 1, 2, \dots, T, \tag{2}$$

against the alternative hypothesis

$$\begin{aligned} H_1 :\ & y_t \sim I(0), \quad t = 1, \dots, k^*, \\ & y_t \sim I(d), \quad t = k^*+1, \dots, T, \ 0 < d < 1.5, \ d \neq 0.5 \end{aligned} \tag{3}$$

where T denotes the largest monitoring sample size and k^* is the unknown change point. We use the following variance ratio statistic to sequentially detect changes from $I(0)$ to $I(d)$ until the time horizon T.

$$\Gamma_T(s) = \frac{\sum_{t=1}^{[Ts]}\left(\sum_{i=1}^{t}\hat{\varepsilon}_i\right)^2}{[Ts]\sum_{t=1}^{[Ts]}\hat{\varepsilon}_t^2}, \tag{4}$$

where $[x]$ denotes the largest integer smaller than x and $\hat{\varepsilon}_i = y_i - \bar{y}_{[Ts]}$ with $\bar{y}_{[Ts]} = \frac{1}{[Ts]}\sum_{i=1}^{[Ts]} y_i$ represents the OLS residuals from the regression of y_i on μ, $i = 1, \dots, [Ts]$.

3. Main results

Theorem 1. *If Assumption 1 holds, then under the null hypothesis H_0 we have that (as $T \to \infty$)*

$$\Gamma_T(s) \Rightarrow (s\sigma_0)^{-2}\int_0^s \left(B(r) - \frac{r}{s}B(s)\right)^2 dr.$$

Proof. Let $t = [Tr]$, by Assumption 1 we have

$$\begin{aligned} T^{-1/2}\sum_{i=1}^{t}\hat{\varepsilon}_i &= T^{-1/2}\sum_{i=1}^{[Tr]} x_i - \frac{[Tr]}{[Ts]}T^{-1/2}\sum_{i=1}^{[Ts]} x_j \\ &\Rightarrow B(r) - \frac{r}{s}B(s) \end{aligned}$$

$$\begin{aligned} T^{-1}\sum_{t=1}^{[Ts]}\hat{\varepsilon}_t^2 &= T^{-1}\sum_{t=1}^{[Ts]}\left(x_t - \frac{1}{[Ts]}\sum_{i=1}^{[Ts]} x_j\right)^2 \\ &= \frac{[Ts]}{T}[Ts]^{-1}\sum_{t=1}^{[Ts]} x_t^2 - \frac{1}{[Ts]}\left(T^{-1/2}\sum_{i=1}^{[Ts]} x_j\right)^2 \\ &\xrightarrow{p} s\sigma_0^2. \end{aligned}$$

Then, the continuous mapping theorem gives that

$$\begin{aligned} \Gamma_T(s) &= \frac{T^{-1}\sum_{t=1}^{[Ts]}\left(T^{-1/2}\sum_{i=1}^{t}\hat{\varepsilon}_i\right)^2}{\frac{[Ts]}{T}T^{-1}\sum_{t=1}^{[Ts]}\hat{\varepsilon}_t^2} \\ &\Rightarrow (s\sigma_0)^{-2}\int_0^s\left(B(r) - \frac{r}{s}B(s)\right)^2 dr. \end{aligned}$$

Theorem 2. *Let Assumptions 1 and 2 hold, then under the alternative hypothesis H_1 we have that*

$$\Gamma_T(s) = O_p(T^{2d}), \quad s > k^*.$$

Proof. According to Assumptions 1 and 2 and the proof of Theorem 1, if there occurs a change from short to long memory at k^* with $k^* = [Tu] < [Ts]$, and $d > 0$, then

$$\begin{aligned} T^{-1/2-d}\sum_{t=1}^{[Ts]} x_t &= T^{-1/2-d}\sum_{j=1}^{k^*} x_j + T^{-1/2-d}\sum_{j=k^*1}^{[Ts]} x_j \\ &\Rightarrow B_d(s) - B_d(u). \end{aligned}$$

On the other hand,

$$\begin{aligned} T^{-1}\sum_{t=1}^{[Ts]}\hat{\varepsilon}_t^2 &= T^{-1}\sum_{t=1}^{k^*} x_t^2 + T^{-1}\sum_{t=k^*+1}^{[Ts]} x_t^2 + o_p(1) \\ &\xrightarrow{p} u\sigma_0^2 + (s-u)\sigma_1^2. \end{aligned}$$

This indicates that if $k^* < [Ts]$, the numerator of the statistic $\Gamma_T(s)$ diverges to infinity at a rate of T^{2+2d}, and the denominator of the statistic $\Gamma_T(s)$ diverges to infinity at a rate of T^2. This completes the proof of Theorem 2.

Theorem 2 implies that we can reject the short memory null hypothesis in favor of a change from short to long memory for large values. Theorem 1 shows that the null distribution of the monitoring statistic $\Gamma_T(s)$ depends on the unknown constant σ_0^2 and the long-run variance, which is hidden behind the Brownian motion $B(\cdot)$. Furthermore, Merlevède et al. (2006) has shown that many types of short memory sequences satisfy Assumption 1, but our previous simulations indicate that different types of innovation processes give very different finite sample critical values. The sample size also has some influence on the critical values. In order to overcome these drawbacks, we use the following sieve bootstrap method to approximate the asymptotic critical values of statistic $\Gamma_T(s)$. Simulations in the next section show that the empirical size can be controlled well via asymptotic critical values.

The steps of sieve bootstrap methodology are constructed as follows:

Step 1. Having observed the samples $y_1, y_2, \dots, y_{[T\tau]}$, compute the OLS residuals $\hat{x}_t = y_t - \frac{1}{[T\tau]}\sum_{i=1}^{[T\tau]} y_i$, and centered residuals $\hat{\varepsilon}_t = \hat{x}_t - \frac{1}{[T\tau]}\sum_{i=1}^{[T\tau]}\hat{x}_i$.

Step 2. We fit an autoregressive processes to the centralized residuals

$$\hat{\varepsilon}_t = \beta_1\hat{\varepsilon}_{t-1} + \beta_2\hat{\varepsilon}_{t-2} + \cdots + \beta_{p(T)}\hat{\varepsilon}_{t-p(T)}$$