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A modified test against spurious long memory☆

Robinson Kruse*

Rijksuniversiteit Groningen, Faculty of Economics and Business, University of Groningen, PO Box 800, 9700 AV Groningen, The Netherlands
CREATES, Aarhus University, Department of Economics and Business, Fuglesangs Allé 4, DK-8210 Aarhus V, Denmark

HIGHLIGHTS

- Detection of spurious long memory can be complicated when the data contains also a truly fractionally integrated component.
- A recent test against spurious long memory by Qu (2011) is modified to improve its finite-sample power properties in this case.
- The modified test builds on prior fractional differencing and offers gains in power while maintaining similar size properties.
- The same critical values can be used as the modified statistic has the same limit distribution as the original test by Qu (2011).

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ABSTRACT

This paper considers the situation where a time series is composed of a fractionally integrated component with long memory parameter $d \in (-1/2; 1/2)$ and some contamination in the form of level shifts or trends. The test against spurious long memory by Qu (2011) is consistent in this case as the standard local Whittle estimator for unknown d is upward biased. As demonstrated in this work, the power can be improved by removing the fractional component from the series prior to application of the test. This task can be accomplished by using the modified local Whittle approach by Hou and Perron (2014). This estimator is robust against contaminations and yields nearly unbiased point estimates of d, irrespective of whether contaminations are present or not. The suggested testing procedure has similar size properties as the original test and is often more powerful.

1. Introduction

Testing for true versus spurious long memory is an active field of econometric research nowadays. Spurious long memory processes are characterized by structural breaks or (smooth) trends and also lead to hyperbolically decaying autocorrelations, see Diebold and Inoue (2001). Recent contributions for tests against true long memory (in the sense of fractional integration) include Ohanissian et al. (2008), Perron and Qu (2010), Qu (2011) and Hassler et al. (2014) inter alia.

Qu (2011) proposes a test in the frequency domain based on the profiled local Whittle likelihood function. While true long memory processes are characterized by a uniform behaviour of the periodogram, structural break processes (as random level shifts) behave non-uniformly around certain frequencies. The suggested test statistic therein exploits this distinct behaviour for discrimination. The test turns out to be powerful when the series is solely generated by random level shifts, non-monotonic trends or Markov regime switching.[1] But recent work find evidence for the presence of both features, see Varneskov and Perron (2015) and Grassi and Santucci di Magistris (2014). This work extends the scope towards mixed processes where both, a fractionally integrated component and some form of contamination are possibly present and thereby follows McCloskey and Perron (2013) and Hou and Perron (2014).

The test by Qu (2011) is still consistent also when the data generating process (DGP) consists of fractional integration and some

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* Correspondence to: CREATES, Aarhus University, Department of Economics and Business, Fuglesangs Allé 4, DK-8210 Aarhus V, Denmark.
E-mail addresses: y.r.kruse@rug.nl, rkruse@creates.au.dk.

[1] See also Leccadito et al. (2015) for a recent comparison of competing tests. Their results indicate that the Qu (2011) test is highly competitive.

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contamination. It turns out that it can be improved in terms of power (while maintaining similar size properties) in such situations by transforming the data according to a GLS-type approach. In fact, it is suggested to take a fractional difference of the series prior to testing. By doing so, the fractionally integrated component is removed. When the true DGP does not contain a fractionally integrated component, the loss in power is small and outweighed by the potential power gains when it is actually present. Even for rather small values of d, power gains are already achievable.

The suggested procedure works as follows: In a first step, the fractional component is filtered from the series. As the true long memory parameter is unknown, some consistent estimator shall be used. The standard local Whittle estimator is known to be strongly upward biased and thus not of much use in this respect. On the contrary, the recently proposed modified local Whittle estimator by Hou and Perron (2014) is robust and can cope with several forms of contaminations, including random level shifts. The simulation results in Hou and Perron (2014) are encouraging and demonstrate that the suggested modification yields nearly unbiased point estimates of d, irrespective of whether the contamination is actually present or not. Thus, the fractionally integrated component can be removed consistently by fractional differencing. As typical for such a GLS-type approach, the contamination is subject to fractional differencing as well. The consequences for the behaviour of the periodogram are discussed. The modified test is consistent as well and it turns out that it is often more powerful. While the original test is introduced in Section 2, suggested modification is presented in Section 3 and further investigated via Monte Carlo simulations in Section 4 where also size and power experiments are conducted.

2. Testing against spurious long memory

This section briefly reviews the Qu (2011) test.[2] Under the null hypothesis, the series y_t $(t = 1, 2, \dots, T)$ is stationary and has spectral density $f(\lambda) \simeq G\lambda^{-2d}$ as $\lambda \to 0_+$. The long memory parameter d is restricted to lie in the interval $(-1/2; 1/2)$ and G is a positive, finite constant. This situation covers for instance the class of ARFIMA processes. The local Whittle likelihood is given by $(m < T)$

$$Q(G, d) = \frac{1}{m}\sum_{j=1}^{m}\left(\log G\lambda_j^{-2d} + \frac{I_{y,j}}{G\lambda_j^{-2d}}\right)$$

where $\lambda_j = 2\pi j/T$ are frequencies and $I_{y,j}$ denotes the periodogram of y_t, i.e.

$$I_{y,j} = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} y_t \exp(i\lambda_j t)\right|^2.$$

Under the validity of the null hypothesis, the expected ratio of the periodogram to the spectral density tends to unity for all values of j, given that $j \to \infty$ when $T \to \infty$ and $j/T \to 0$. The profiled likelihood (after concentrating G out of Q) reads

$$R(d) = \log G(d) - \frac{2}{m} d \sum_{j=1}^{m} \log \lambda_j$$

with $G(d) = \frac{1}{m}\sum_{j=1}^{m} \lambda^{2d} I_{y,j}$. The test statistic by Qu (2011) is based on the first derivative of R with respect to d and is related to the Lagrange Multiplier test by Robinson (1994). It is given by

$$W = \sup_{r\in[\varepsilon,1]} \frac{1}{\sqrt{\sum_{j=1}^{m} \upsilon_j^2}} \left|\sum_{j=1}^{[mr]} \upsilon_j \left(\frac{I_{y,j}}{G(\widehat{d})\lambda_j^{-2\widehat{d}}} - 1\right)\right| \qquad (1)$$

with $\upsilon_j = \log\lambda_j - \frac{1}{m}\sum_{j=1}^{m}\log\lambda_j$. Here, $\widehat{d}$ signifies the standard local Whittle estimator and ε is a trimming parameter.[3] The limiting distribution of W is non-standard and critical values are provided in Qu (2011). Under the alternative, the series y_t is short memory, but subject to some form of contamination. The probably most prominent example is the case of random level shifts, see for instance Qu and Perron (2013). In this case, $y_t = x_t + u_t$ with x_t being short memory and $u_t = \sum_{i=1}^{t}\delta_i = \sum_{i=1}^{t}\pi_i\eta_i$ with $\pi_t \sim i.i.d.\ B(p/T, 1)$ and $\eta_t \sim i.i.d.\ (0, \sigma_\eta^2)$; π_t and η_t are mutually independent. Perron and Qu (2010) show that the periodogram of y_t can be decomposed as follows:

$$I_{y,j} = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} x_t \exp(i\lambda_j t)\right|^2 + \frac{1}{2\pi T}\left|\sum_{t=1}^{T} u_t \exp(i\lambda_j t)\right|^2 + \frac{2}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T} x_t u_t \cos(\lambda_j(t-s)).$$

For $\lambda_j = o(1)$, they derive the following behaviour

$$I_{y,j} = O_p(1) + O_p\left(\frac{1}{T\lambda_j^2}\right) + O_p\left(\frac{1}{\sqrt{T}\lambda_j}\right).$$

The second term dominates $I_{y,j}$ for $j = o(\sqrt{T})$, while the first term is dominant for $j/\sqrt{T} \to \infty$. The consistency of the W test hinges on the upward bias of the standard local Whittle estimator $\widehat{d}$ (Theorem 1 Qu, 2011). Even if the series is composed by both types (stationary fractional integration with parameter $d \in (-1/2; 1/2)$ and contamination), the test retains its consistency. An important, but mild requirement is that the standard local Whittle estimator is still upward biased, i.e. $P(\widehat{d} - d > \epsilon) \to 1$ for $\epsilon > 0$ (Corollary 1 Qu, 2011) and that a variance condition holds. Following McCloskey and Perron (2013), the periodogram behaves as

$$I_{y,j} = O_p\left(\frac{1}{\lambda_j^{2d}}\right) + O_p\left(\frac{1}{T\lambda_j^2}\right) + O_p\left(\frac{1}{\sqrt{T}\lambda_j^{1+d}}\right).$$

The second component dominates for $j = o\left(T^{(1-2d)/(2-2d)}\right)$, while the first one dominates for $jT^{(2d-1)/(2-2d)} \to \infty$. For $d = 0$, the former type of behaviour emerges as a special case, see Perron and Qu (2010).

3. Modified test

3.1. Infeasible version

This section introduces the suggested modification. First, an infeasible version is presented to illustrate the idea. Below, the implementation of a feasible test procedure is described. There are two steps to be considered. Let L denote the lag operator.

1. Build the fractional difference $z_t = (1-L)^d y_t$ for the observable time series y_t.
2. Apply the Qu (2011) test statistic to z_t (instead of y_t). In particular, compute

$$W_z = \sup_{r\in[\varepsilon,1]} \frac{1}{\sqrt{\sum_{j=1}^{m} \upsilon_j^2}} \left|\sum_{j=1}^{[mr]} \upsilon_j \left(\frac{I_{z,j}}{G_z(\widehat{d}_z)\lambda_j^{-2\widehat{d}_z}} - 1\right)\right| \qquad (2)$$

where $\widehat{d}_z$ denotes the standard local Whittle estimator for d applied to the series z_t. Moreover, $G_z(\widehat{d}_z) = \frac{1}{m}\sum_{j=1}^{m}\lambda^{2\widehat{d}_z} I_{z,j}$.

[2] The reader is referred to the original contribution for a complete presentation.

[3] Qu (2011) recommends $m = T^{0.7}$ and $\varepsilon = 0.02$ for $T \geq 500$.