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Price vs. quantity competition in vertically related markets. Generalization

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HIGHLIGHTS

- Generalization of the result on wholesale prices illustrated in Alipranti et al. (2014).
- Wholesale prices are above marginal costs if retailers compete in prices.
- Wholesale prices are below marginal costs if retailers compete in quantities.

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ABSTRACT

This paper generalizes the result of Alipranti et al. (2014) regarding the wholesale prices: I prove that upstream firms always charge the wholesale prices above (below) their marginal costs in case of Bertrand (Cournot) competition downstream. Alipranti et al. (2014) demonstrates this result for the case of linear demand functions and monopolist that sells its product to two retailers. I relax the assumption of linear demands, allow for arbitrary number of retailers and for the competition upstream.

1. Introduction

The recent paper Alipranti et al. (2014) shows that (contrary to the case of direct sales) in a vertically related market Cournot competition is preferred to Bertrand one in terms of welfare. This result is explained by the fact that in case of quantity competition downstream the manufacturer sets wholesale prices at lower levels than in case of price competition downstream.

The analysis in Alipranti et al. (2014) is conducted under the assumptions of linear demand functions, monopoly upstream and two retailers.

The aim of this note is to generalize the result on wholesale prices. I prove that for arbitrary number of retailers, for the possibility of competition upstream and for the general form of the demand functions[1] upstream producers always charge the wholesale prices above (below) their marginal costs in case of Bertrand (Cournot) competition downstream. In other words, the result on wholesale prices is generalized in three ways: Firstly, the demands have general forms (instead of linear ones considered in Alipranti et al., 2014). Secondly, an arbitrary number of retailers is assumed (instead of just two). Thirdly, the competition upstream is allowed.

2. Model

There are $K \geq 1$ upstream firms that produce substitute products at constant marginal costs c. Each upstream firm sells its good to $M \geq 1$ downstream firms. It is assumed that $\max(K, M) \geq 2$. In total there are $N = M \cdot K$ downstream producers. Downstream firms do not have other costs than spending on the input from upstream enterprises.

So, I consider an exclusive contracts model: each upstream firm i has exclusive contracts with M firms downstream. Notice that if $K = 1$ we are in the situation of Alipranti et al. (2014) market structure with arbitrary number of firms downstream (instead of just 2). If $M = 1$, then the framework reduces to the case of competing vertical structures.

The timing is as follows: at the first stage of the game every upstream producer $k = 1, \ldots, K$ simultaneously and separately bargains with each of its retailer $m = 1, \ldots, M$ over the components

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[1] For substitute products.

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of the two-part tariff contract, that is over a wholesale price, w^{km}, and a fixed fee, F^{km}.[2] At the second stage downstream firms simultaneously choose prices or quantities (depending on the competition mode) after observing each other's contract terms.

The bargaining is modeled in the same way as in Alipranti et al. (2014): it is assumed that each upstream firm simultaneously and independently solves the Nash bargaining problem with each of "its" retailer. In each of these Nash bargaining problems upstream manufacturer has bargaining power β, while downstream producer's bargaining power is $1-\beta$, with $\beta \in (0, 1]$.

2.1. Consumer side

The utility of the consumer depends on the goods sold by the retailers and expenditures on composite commodity (i.e. T):

$$U = U(q^1, q^2, \dots, q^N) + T, \tag{1}$$

where q^i, $i = 1, 2, \dots, N$ is the quantity of commodity i consumed.[3]

Assumption 0. The quantities of all the goods enter symmetrically the utility function: $U(q^1, \dots q^{h-1}, q^h, q^{h+1}, \dots, q^{i-1}, q^i, q^{i+1}, \dots, q^N) = U(q^1, \dots q^{h-1}, q^i, q^{h+1}, \dots, q^{i-1}, q^h, q^{i+1}, \dots, q^N) \forall h, i$.

Assumption 1. $\frac{dU}{dq^i} > 0$; $\frac{d^2U}{d(q^i)^2} < 0$, $i = 1, \dots, N$.

Assumption 2. $\frac{d^2U}{dq^i dq^j} < 0$, $\forall i, j = 1, \dots, N$; $i \neq j$.

Assumption 2 is natural for substitutes.

From the utility maximization problem [$\max_{q^1, q^2, \dots, q^N} U(q^1, q^2, \dots, q^N) + T$ s.t. $p^1q^1 + p^2q^2 + \dots + p^Nq^N + T \le I$, where I is the income of consumer and p^i is a unitary price of good i] we get inverse demands for goods $1, \dots, N$:

$$\begin{cases} p^1 = \dfrac{dU}{dq^1} = D^1(q^1, q^2, \dots, q^N) \\ p^2 = \dfrac{dU}{dq^2} = D^2(q^1, q^2, \dots, q^N) \\ \dots \\ p^N = \dfrac{dU}{dq^N} = D^N(q^1, q^2, \dots, q^N). \end{cases} \tag{2}$$

Assumption 3. $D^i_i = \frac{dD^i}{dq^i} < D^i_j = \frac{dD^i}{dq^j}$.

Lemma 1. $D^i_i < 0$; $D^i_j < 0$.

Proof. Differentiation of (2) with respect to $q^1, q^2, \dots, q^N$; and application of Assumptions 1, 2 give the result of Lemma 1. □

From (2) we obtain direct demands for the goods:

$$\begin{cases} q^1 = Q^1(p^1, p^2, \dots, p^N) \\ q^2 = Q^2(p^1, p^2, \dots, p^N) \\ \dots \\ q^N = Q^N(p^1, p^2, \dots, p^N). \end{cases} \tag{3}$$

Lemma 2. $Q^i_i = \frac{dQ^i}{dp^i} < 0$; $Q^i_j = \frac{dQ^i}{dp^j} > 0$ *and* $|Q^i_i| > (N-1)Q^i_j$ $\forall j \neq i$.

Proof. Differentiating (2) with respect to p^f, $f = 1, \dots, N$ and taking into account Assumption 0 in a symmetric equilibrium we get:

$$\begin{cases} \dfrac{dq^i}{dp^i} = Q^i_i = \dfrac{D^j_j + (N-2)D^j_i}{D^i_i \cdot (D^j_j + (N-2)D^j_i) - (N-1)D^i_j \cdot D^j_i} \\ \dfrac{dq^i}{dp^j} = Q^i_j = \dfrac{-D^i_j}{D^i_i \cdot (D^j_j + (N-2)D^j_i) - (N-1)D^i_j \cdot D^j_i}. \end{cases} \tag{4}$$

Due to Lemma 1 and Assumption 3 the result of Lemma 2 follows. □

Let $p = (p^1, \dots, p^N)$, $q = (q^1, \dots, q^N)$, $w = (w^1, \dots, w^N)$.[4] Then the inverse demand function for good i is $p^i = D^i(q)$. The direct demand function for good i is $q^i = Q^i(p)$.

2.2. Bertrand competition

In case of price competition among retailers the payoff function of retailer i is $R^{Bi} = (p^i - w^i)Q^i(p) - F^i$.

Let subscripts to R^{Bi} denote the partial derivatives with respect to prices. For example, $R^{Bi}_{ij} = \frac{dR^{Bi}}{dp^i dp^j}$.

Assumptions 4 and 5 guarantee stability of the equilibrium in prices and that the prices are strategic complements.

Assumption 4 (*Stability*)**.** For any w and $i, j = 1, \dots, N; i \neq j$ $R^{Bi}_{ii}(p(w), w^i) + (N-1)R^{Bi}_{ij}(p(w), w^i) < 0$.[5]

Assumption 5 (*Prices are Strategic Complements*)**.** For all $p : p^i \ge w^i$ $R^{Bi}_{ij}(p(w), w^i) = Q^i_j(p) + (p^i - w^i)Q^i_{ij} > 0$.

2.3. Cournot competition

In case of quantity competition among retailers the payoff function of retailer i is $R^{Ci} = (D^i(q) - w^i)q^i - F^i$.

Again subscripts to R^{Ci} denote the partial derivatives with respect to quantities. For example, $R^{Ci}_{ij} = \frac{dR^{Ci}}{dq_i dq_j}$.

Assumption 6 (*Stability*)**.** For any w and $i, j = 1, \dots, N; i \neq j$ $R^{Ci}_{ii}(q(w), w^i) + |R^{Ci}_{ij}(q(w), w^i)| < 0$.[6]

Assumption 7 (*Quantities are Strategic Substitutes*)**.** For all $q : q^i \ge 0$ $R^{Ci}_{ij}(q(w), w^i) = D^i_j(q) - w^i + q_i D^i_{ij} < 0$.

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2 There is a simultaneous bargaining over N wholesale prices and fixed fees. Further in the text w and F have just one superscript indicating the retailer that participated in the bargaining process. For example, the components of the two-part tariff contract between upstream producer k and retailer i are w^i and F^i.

3 It is also the quantity of the good sold by retailer i.

4 In case of Bertrand (Cournot) competition downstream p, q, w will have superscript $B(C)$.

5 See Leahy and Neary (1997, Lemma 1), for the formulation of stability condition for the case of N-firm symmetric oligopoly.

6 Due to Assumption 7 and the fact that second-order condition must hold at the equilibrium point, the expression $R^{Ci}_{ii}(q(w), w^i) + (N-1)R^{Ci}_{ji}(q(w), w^i)$ (taken from Leahy and Neary (1997, Lemma 1)) is always negative. Linearization of the system of first-order conditions around the equilibrium point and calculation of eigenvalues of the coefficient matrix allows to conclude that all eigenvalues are negative iff $R^{Ci}_{ii}(q(w), w^i) + |R^{Ci}_{ji}(q(w), w^i)| < 0$. This condition coincides with Assumption 1 in Leahy and Neary (1997).