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Contests, private provision of public goods and evolutionary stability

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HIGHLIGHTS

- The private provision of public goods can be incentivized by a contest.
- Evolutionary stability of public goods game with contest is studied.
- If evolutionary stability and Nash equilibrium coincide, the outcome is efficient.

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ABSTRACT

We study evolutionary stability for public goods games incentivized by a contest. In a quasi-linear setting, we derive conditions such that evolutionary stability, Nash equilibrium and efficient solution coincide.

1. Introduction

Contests and lotteries can be helpful devices to incentivize the private provision of public goods (Morgan, 2000; Kolmar and Wagener, 2012). Adding to a situation where a public good is provided through private contributions a contest that rewards higher contributions by improving the contributor's chances of winning a rent or prize can alleviate the under-provision problem for public goods. Under the assumption of Nash play, a suitable combination may even implement an efficient level of the public good (Kolmar and Wagener, 2012).

We study how contests and the private provision of public goods work together in an (direct) evolutionary framework. We analyze finite-player evolutionarily stable strategies (ESS) of a quasi-linear private provision game for a public good (henceforth: public goods game) that is combined with a contest where individual success increases in one's contributions to the public good. We relate the ESS to the efficient solution and to the Nash equilibrium of that game and study conditions such that the outcomes coincide. Our results are as follows:

- The finite-player ESS in the provision-with-contest game is identical to the ESS of the contest alone and, thus, depends on the prize and the contest success function only (Result 1). The ESS predicts the outcome when players care about their relative, rather than about absolute, payoffs. As the public goods game with contest is a generalized aggregative game, its ESS is globally stable and stochastically stable. I.e., knowing the ESS shortcuts the full study of specific dynamic processes of learning, imitation, reproduction etc.
- The motive of spite inherent in evolutionary play exacerbates both the under-provision in the public goods game and the over-investment problem in the contest. By balancing these opposing trends, a suitable combination of contest and public goods game, characterized in Result 2, implements the efficient level of the public good in evolutionary play.
- With Tullock contests, the conditions on prize and contest success function such that the efficient level of the public good is provided turn out to be same for Nash and for evolutionary

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play. They establish a trade-off between the value of the prize and contest decisiveness (Result 3).
- Generally, if Nash equilibrium and ESS coincide in a public goods game with contest, they both implement the efficient level of the public good (Result 4). The necessary condition trades off the value of the prize and the negative spillovers in the contest (Result 5).

2. Public goods game with a contest

Notation. We consider symmetric normal form games with $n \geq 2$ identical players, indexed by i. Each player's strategy set is given by a closed interval $[0, m]$ for some $m > 0$. Vectors in $\mathbb{R}^n$ are denoted in bold-face: $\mathbf{x} = (x_1, \dots, x_n)$. We write $\mathbf{1} = (1, \dots, 1)$. Symmetric vectors are recognizable by superscripts or other adornments. E.g., for scalar x^E we denote by $\mathbf{x}^E = x^E \cdot \mathbf{1}$. We write $\mathbf{x}_{-i}$ for the vector of all x_j except for x_i.

Goods. Endowed with an amount $m > 0$ of a dual-use good, each individual decides how much of it she spends on contributing to a public good ($x_i \in [0, m]$) or to consume directly ($m - x_i$). The provision level, g, of the public good equals the sum of individual contributions: $g = g(\mathbf{x}) = \sum_{i=1}^n x_i$. There is no way to provide the public good other than by players' contributions x_i. In particular, the contest designer (see below) is not able to add to, or subtract from, the public good.

Contest. To incentivize contributions to the public good, a contest is installed. With probability p_i it pays a prize of value $z > 0$ to individual i. The prize is financed by equal lump-sum taxes z/n and measured in terms of private consumption. Winning probabilities p_i are determined by a contest success function (CSF) that depends on own contributions, x_i, and an aggregate, $h(\mathbf{x})$, of all contributions:

$$p_i = p(x_i, h(\mathbf{x})). \tag{1}$$

The CSF has standard properties: it is a probability on the set of agents ($0 \leq p(x, h(\mathbf{x})) \leq 1$ and $\sum_{i=1}^n p(x_i, h(\mathbf{x})) = 1$ for all $\mathbf{x} \in \mathbb{R}^n_+$). It strictly increases in x and strictly decreases in h; the aggregate $h(\mathbf{x})$ strictly increases in all its arguments. For simplicity, p and h are twice differentiable. Moreover, $h(\mathbf{x})$ is symmetric, i.e., $h(\pi(\mathbf{x})) = h(\mathbf{x})$ for all permutations π of $\mathbf{x}$. The latter assumption implies that, for all $\mathbf{x}$ and whenever $x'_i = x'_j$,

$$p(x'_i, h(\mathbf{x})) = p(x'_j, h(\mathbf{x})) \quad \text{and} \quad \frac{\partial\, p(x'_i, h(\mathbf{x}))}{\partial\, x_i} = \frac{\partial\, p(x'_j, h(\mathbf{x}))}{\partial\, x_j}.$$

From (1), the marginal effect of own contributions on an individual's winning probability,

$$\frac{dp_i}{dx_i} = \frac{d}{dx_i} p(x_i, h(\mathbf{x})) = \frac{\partial p(x_i, h(\mathbf{x}))}{\partial x_i} + \frac{\partial p(x_i, h(\mathbf{x}))}{\partial h} \frac{\partial h(\mathbf{x})}{\partial x_i},$$

can be decomposed into two parts. The (positive) "aggregate-taking" effect $\frac{\partial p(x_i, h(\mathbf{x}))}{\partial x_i}$ measures how an increase in one's contribution raises the odds, given that the aggregate, $h(\mathbf{x})$, of all contributions remains unchanged. The (negative) "aggregate-changing" effect $\frac{\partial p(x_i, h(\mathbf{x}))}{\partial h} \frac{\partial h(\mathbf{x})}{\partial x_i}$ measures how increasing x_i decreases i's winning chances via a raise in aggregate contributions.

Example: Tullock CSF. A prominent class of CSFs that satisfies our assumptions are Tullock CSFs:

$$p(x_i, h(\mathbf{x})) = x_i^r / h(\mathbf{x}) \quad \text{with } h(\mathbf{x}) := \sum_{j=1}^n x_j^r. \tag{2}$$

Parameter $r > 0$ measures the decisiveness of the contest. For $r = 1$, the contest is a lottery. To avoid technical problems we assume $r \leq n/(n-1)$ whenever we discuss Tullock contests.

Preferences. Individuals have quasi-linear preferences over the (expected) consumption of the private good, $m - z/n - x_i + p_i z$, and the public good:

$$\begin{aligned} u_i &= m - z/n - x_i + p_i z + v(g) \\ &= m - z/n - x_i + p(x_i, h(x_i, \mathbf{x}_{-i})) \cdot z + v(g(x_i, \mathbf{x}_{-i})) \\ &= u(x_i, \mathbf{x}_{-i}). \end{aligned}$$

Function v is strictly increasing and strictly concave: $v'(g) > 0 > v''(g)$. We assume $v'(0) > 1 > nv'(nm - z)$ for all z we consider. These inequalities preclude that all resources should optimally be devoted to, respectively, the private consumption or the public good. Writing u_i as

$$U(x_i, \mathbf{x}) = m - z/n - x_i + p(x_i, h(\mathbf{x}))z + v(g(\mathbf{x})), \tag{3}$$

shows that the public goods game with a contest is a generalized aggregative game: for each player, payoffs depend on the own action, x_i, and on symmetric aggregates, h and g, of all strategies, $\mathbf{x}$.

3. Efficiency and Nash equilibrium

Efficiency. By the separability and the strict concavity of v, the efficient[1] level of the public good, g^*, is uniquely given by the Samuelson condition:

$$nv'(g^*) = 1 \quad \text{or} \quad g^* = v'^{-1}(1/n). \tag{4}$$

Denote by $x^* := g^*/n$ the contribution that, if made by everybody, efficiently provides the public good: given available resources, $\mathbf{x}^* = x^* \mathbf{1}$ is the symmetric solution to the problem $\max_{\mathbf{x}} \sum_j u_j(\mathbf{x})$. It satisfies

$$\sum_{j=1}^n \frac{\partial\, u_j(\mathbf{x}^*)}{\partial\, x_i} = 0 \quad \text{for all } i. \tag{5}$$

Nash equilibrium. A symmetric Nash equilibrium is a contribution level, x^N, such that $u(\mathbf{x}^N) \geq u(x_i, \mathbf{x}^N_{-i})$ for all $x_i \in [0, m - z/n]$. In the public goods game with contest, it satisfies

$$-1 + v'(nx^N) + z \cdot \frac{d}{dx_i} p(x_i^N, h(\mathbf{x}^N)) = 0. \tag{6}$$

Efficiency and Nash play. Without contest, the provision of the public good is inefficiently low: $v'(nx^N) > v'(nx^*)$ for $p = 0$ or $z = 0$. Adding a contest may remedy this. Combining (4) and (6), efficiency requires that the CSF locally satisfies

$$z \cdot \frac{dp(x_i^*, h(\mathbf{x}^*))}{dx_i} = \frac{n-1}{n}. \tag{7}$$

(Kolmar and Wagener, 2012, Eq. (13)). Condition (7) indicates that a contest designer can trade off a sharper decisiveness (greater sensitivity of p_i with respect to x_i) for a larger prize.

4. Evolutionary stability

A strategy x^E is called an evolutionarily stable strategy (ESS) if

$$u(x^E, x, \underbrace{x^E, \dots, x^E}_{n-2}) \geq u(x, \underbrace{x^E, \dots, x^E}_{n-1})$$

for all $x \geq 0$. An ESS is a strategy that, when played by all players, cannot be invaded by single mutations: any deviating player earns a lower payoff than the non-deviating players (relative payoff comparison).

[1] The efficient level solves

$$\max_{g,(c_i)_{i=1,\dots,n}} \sum_i (c_i + v(g)) \quad \text{s.t.}\ g + \sum_i c_i \leq mn - z,$$

where the right-hand side denotes available resources after financing the prize.