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Maximum likelihood estimation of the revenue function system with output-specific technical efficiency

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HIGHLIGHTS

- We model output-specific technical efficiency (OSTE) in a revenue maximizing model.
- We show that the translog revenue share system with OSTE follows a closed skew-normal distribution (CSN).
- We use the CSN results to derive the log-likelihood function for ML estimators of the parameters and predictor of OSTE.

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ABSTRACT

In this paper we propose a non-radial and output-specific measure of technical efficiency which is new in a stochastic frontier model. We consider a multi-output multi-input transformation function formulation that incorporates output-specific technical efficiency (OSTE) in a revenue maximizing framework. Starting from the dual revenue function with OSTE, we develop the maximum likelihood method to estimate the parameters of the translog revenue-share system as well as predict OSTE components using some novel results from the closed skew-normal distribution.

1. Introduction

In this paper we consider a multi-output multi-input transformation function and introduce a new measure of efficiency, in particular, non-radial output-specific technical efficiency (OSTE) in the stochastic frontier framework. We consider a revenue maximizing model in which the revenue function inherits the OSTE terms from the transformation function. The OSTE terms are transmitted to the translog revenue share functions derived from the translog revenue function using the duality results. We develop the maximum likelihood method to estimate the parameters of the revenue-share system as well as predict the OSTE components using some novel results from the closed skew-normal distribution. The formulation proposed in the paper generalizes the multiplicative general error model in the spirit of Kumbhakar and Tsionas (2011) to accommodate (i) multiple outputs in a revenue maximizing framework and (ii) OSTE in the production technology. Although the closed skew-normal distribution has been used before in the efficiency literature in a single equation stochastic frontier (SF) model (e.g., Colombi et al. (2014), Chen et al. (2014) and Filippini and Greene (forthcoming)), our approach is unique because our SF model uses a system of equations and we predict a vector of OSTE (one for each output) using cross-sectional data.[1] There are some non-parametric directional distance function papers (for example, Simar and Vanhems (2012) and Simar et al. (2012)) that are

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[1] We presented the model in a cross-sectional framework. However, it can be generalized to accommodate panel data. We plan to do it in a future paper.

used to estimate input and output inefficiencies (slacks), instead of fixing the direction *a priori*. These directional distance function papers do not include noise directly as we do in stochastic frontier and therefore there is no obvious problem of separating noise from inefficiency. Also our approach uses formal economic behavior to estimate the slacks.[2]

Our starting point is a multi-output multi-input transformation function which is generalized to accommodate multiplicative OSTE. We derive the dual revenue function therefrom and the corresponding dual revenue share functions in which the OSTE terms appear. The transformation function with OSTE in a cross-sectional setup is written as[3]

$$F(\theta \odot y, x) \equiv F(y^*, x) = 1 \tag{1}$$

where y is a vector of M outputs, x is a vector of J inputs, $\odot$ represents Hadamard product (element-wise multiplication) and θ is the OSTE vector. To relate the elements of θ to efficiency, we assume that $\theta_m \geq 1, \forall m$. That is, $y_m^* \equiv \theta_m y_m$ is the potential output whereas y_m is the actual (observed) output. Thus, technical efficiency in the production of y_m (OSTE for the mth output) is $y_m/y_m^* = 1/\theta_m \leq 1$ and $\ln\theta_m$ is a measure of inefficiency (slack) in the production of output y_m. Similar interpretation applies to other outputs. We assume θ_m to be i.i.d. random variables which make OSTE observation-specific. If $\theta_m = \theta \, \forall m$ then we have the radial output inefficiency that is widely used in the efficiency literature (see, e.g., Kumbhakar et al. (2015)). Further, if there is only one output, $1/\theta$ is output-oriented technical efficiency.

We assume that firms maximize revenue[4] so that the problem is:

$$\text{Max} \sum_m p_m y_m \quad \text{subject to } F(y^*, x) = 1.$$

The first-order conditions (FOCs) of the above problem are:

$$\frac{p_m}{p_1} = \frac{F_m(.)}{F_1(.)}\frac{\theta_m}{\theta_1}, \quad m = 2, \ldots, M \Longrightarrow \frac{p_m}{p_1} \div \frac{\theta_m}{\theta_1}$$
$$\equiv \frac{p_m^*}{p_1^*} = \frac{F_m(.)}{F_1} \tag{2}$$

where $p_m^* = p_m/\theta_m, \ m = 1, \ldots, M$.

The above $(M-1)$ FOCs in (2) along with the transformation function in (1) can be solved for $y_m^* = \psi_m(p, x), \ m = 1, \ldots, M$. We use these solutions to define the pseudo revenue function $R^*(p, x) = \sum_m p_m^* \psi_m(p, x)$. The advantage of defining R^* is that we can use the envelope theorem (equivalent of Hotelling's lemma) to derive the pseudo output supply functions, $y_m^* = \frac{\partial R^*}{\partial p_m^*}$. Furthermore, we can relate unobserved R^* to observed revenue $R = \sum p_m y_m = \sum_m p_m^* y_m^* = R^*$. That is, $R^* = R$.

We start from a logarithmic form of R^*, i.e., a translog form of $\ln R^*$ and use the envelope theorem to derive the revenue share functions. These are: $\partial \ln R^*/\partial \ln p_m^* = p_m^* y_m^*/R^* = p_m y_m/R \equiv RS_m, m = 1, \ldots, M$. Since the $(M-1)$ shares are independent (the sum of shares being unity), the $(M-1)$ revenue share equations and the revenue function constitute the complete system with M equations and M endogenous output variables.

If the translog revenue function is expressed as

$$\ln R = \beta_0 + \sum_m \beta_m \ln p_m^* + \sum_j \alpha_j \ln x_j + 0.5 \sum_m \sum_n \beta_{mn} \ln p_m^* \ln p_n^*$$
$$+ \sum_m \sum_j \gamma_{mj} \ln p_m^* \ln x_j + 0.5 \sum_j \sum_k \delta_{jk} \ln x_j \ln x_k + v_0 \tag{3}$$

the corresponding revenue share equations are

$$RS_m = \beta_m + \sum_n \beta_{mn} \ln p_n^* + \sum_j \gamma_{mj} \ln x_j + v_m \tag{4}$$

where v_0 and v_m are the noise terms added to the revenue and the revenue share equations. Since the revenue function is homogeneous of degree 1 in output prices, the revenue shares can be rewritten (after imposing the homogeneity constraints) as

$$RS_m = \beta_m + \sum_n \beta_{mn} \ln \tilde{p}_n + \sum_j \gamma_{mj} \ln x_j + \zeta_m + v_m,$$
$$m = 2, \ldots, M, \tag{5}$$

where $\zeta_m = -\sum_n \beta_{mn} \ln\theta_n$ and $\tilde{p}_n = p_n/p_1, \ n = 2, \ldots, M$. It is clear from (5) that ζ_m contains the OSTE terms which are separated from the noise terms v_m. That is, the revenue share system in (5) is the system counterpart of a stochastic frontier model which contains both inefficiency and noise terms. That is, the error terms in the revenue share equations in (5) are composed of noise and OSTE components, which is following the SF literature can be labeled as the composed error vector. The neoclassical revenue shares are obtained from (5) by setting $\zeta_m = 0 \, \forall m$ thereby meaning that firms are fully efficient in producing all the outputs (i.e., $\theta_m = 1 \, \forall m$). We consider estimating $\ln\theta_m$ from (5) although for interpretation we go back to the transformation function in (1).

2. Estimation of the revenue system

2.1. The closed skew-normal distribution

Before any discussion on the likelihood function of the revenue share system in (5), we first introduce the closed skew-normal distribution, which is shown to be the distribution of the revenue system. It has been well established in the statistics literature[5] that the CSN distribution has some good statistical properties similar to the multivariate normal distribution, such as a linear combination of CSN random vectors follows a CSN distribution. In particular, we show in Section 2.3 that it is much easier to derive the predictor of OSTE using the moment generating function of the CSN distribution. Below we use $\phi_q(\cdot; \nu, \Delta)$ and $\phi_q(\cdot; \nu, \Delta)$ to denote the probability density function (pdf) and cumulative distribution function (cdf) of a q-variate multivariate normal distribution with mean ν and variance matrix Δ, and 0_q to denote the $q \times 1$ vector of zeros. Following González-Farías et al. (2004), the closed skew-normal (CSN) random vector is defined as below:

Definition. A random vector ε has a closed skew-normal distribution, denoted as $\varepsilon \sim CSN_{p,q}(\mu, \Sigma, D, \nu, \Delta)$, if it has the pdf

$$f_\varepsilon(\varepsilon) = \frac{\phi_p(\varepsilon; \mu, \Sigma)}{\phi_q(0_q; ; \nu, \Delta)} \Phi_q(D(\varepsilon - \mu); \nu, \Delta).$$

Moreover, its corresponding moment generating function (mgf) is

$$M_\varepsilon(t) = \frac{\Phi_q(D\Sigma t; \nu, \Delta + D\Sigma D^{\mathrm{T}})}{\Phi_q(0_q; \nu, \Delta + D\Sigma D^{\mathrm{T}})} e^{t^{\mathrm{T}}\mu + \frac{1}{2}t^{\mathrm{T}}\Sigma t}, \quad \text{where } t \in \mathbb{R}^p.$$

2 Although in the present model we consider only output slacks, the formulation can be extended to accommodate both input and output slacks in a profit maximizing model.

3 We skipped the observation subscript throughout the paper to avoid notational clutter.

4 As we mentioned earlier, to our knowledge, there is no stochastic frontier model that estimates input and/or output slacks. Our approach in a profit maximizing setup can be used to disentangle both input and output slacks as in Simar and Vanhems (2012) and Simar et al. (2012).

5 For instance, see González-Farías et al. (2004) for a detailed discussion on the properties of CSN.