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Giving Gini direction: An asymmetry metric for economic disadvantage

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HIGHLIGHTS

- Agendas for inequality are usually concerned with too many people on low incomes.
- A new metric, the entropic v, reflects distributional aspirations as seen by the subjects.
- The entropic v utilises the double smoothing property of left and right entropic shifts.
- The entropic v can be given a dollar redistribution dimension and is straightforward to compute.
- In conjunction with the Gini coefficient it can summarise both dispersion and skewness.

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ABSTRACT

A high Gini coefficient could signal either dispersion or else skewness, often of more social concern. Supplementary metrics such as the Atkinson index diagnose the asymmetry with preassigned parameters that reflect user social values. An alternative is proposed that reflects redistribution aspirations as they might be seen by the subjects themselves. The resulting v-metric has a dollar redistribution dimension. The metric, essentially a form of double averaging, can be simplified in terms of means of the left and right entropic distribution shifts, with the partition entropy reflecting implied partition into incomes above or incomes below.

1. Introduction

The social importance of income or wealth distribution has over the years motivated an extensive literature on summary metrics. The Gini coefficient ('Gini'), generated by comparing the progressive income shares with their ownership proportions, remains the most widely cited, but has a number of shortcomings. Some of these are more technical in nature, such as the lack of additive decomposability across subgroups (Allison, 1978). However it is probably fair to say that most of the doubts arise because a given Gini number compounds two effects, one of simple dispersion (spread); and the other of the relative weights attributed to lower versus higher income bands, where positive skewness is a primary object of social concern. An agenda in the latter case has been to find metrics that have economic meaning, as distinct from the textbook third order moment. Alternative or supplementary indexes, such as those of Atkinson (1970) or Generalised Entropy (Pielou, 1966; Shorrocks, 1980), compare distributions over time or location in terms of a user assigned inequality aversion parameter (ε or α). Results are commonly tabulated against different values of ε, with higher ε values as a focus for social concerns about the share of lower income groups.

Approaches of this kind could be regarded as imposing observer value judgements on the choice of metric. However it is also possible to imagine a different thought experiment that seeks to aggregate in some meaningful way how each subject thinks about his or her own income in comparison to that of others. A

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simple way to do this is a linear expected utility scale in which each individual derives positive utility to the extent that his or her income exceeds the conditional expected income below; and negative to the extent that it falls short of the conditional expected income above. The net difference is then aggregated over the relative number in each income band, i.e. the density of the income distribution. A negative index means that on the average people think that others are better off than themselves; so the 'v-index' or metric could evoke net di<u>v</u>ergence, disad<u>v</u>antage or even en<u>v</u>y. As a supplement, the metric enables the observer to tell at a glance whether a higher Gini arises from spread or positive asymmetry.

The resulting metric, essentially a form of double averaging, is easy to compute in terms of the difference between the means of the left and right unit entropic shifts of the original distribution (Bowden, 2012). In turn, these draw on partition entropy, reflecting in this context the distributional weight below or above any given income. From the economic point of view, the v-index can be interpreted as a notional dollar transfer that would be required to restore a symmetric income distribution with the same overall Gini coefficient.

The scheme of development is as follows. Section 2 establishes the metric and its motivation, while Section 3 illustrates with US data, with some further comments on computation.

2. The metric and its motivation

A synthetic approach is adopted, in the first instance with a continuous income distribution. The upper case Y will indicate income considered as a random variable. Without any significant loss of generality, the range of Y can be taken for expositional purposes as $0 \le y < \infty$, with distribution function $F(y)$ and density $f(y)$. A two stage development follows, starting with the formulation of the criterion and then its solution in terms of left and right entropic shifts of F and their means.

For a given income y, define the left and right conditional expected means with respect to F as

$$\mu_l(y) = E_F[Y \mid Y \le y] = \frac{1}{F(y)} \int_0^y x f(x) dx \tag{1a}$$

$$\mu_r(y) = E_F[Y \mid Y > y] = \frac{1}{1 - F(y)} \int_y^\infty x f(x) dx. \tag{1b}$$

Now imagine I have income y. I look at the average income below me: $E[Y \mid Y \le y] = \mu_l(y)$, so relative to this group I am better off to the extent of the difference $(y - \mu_l(y))$. Then I look at the average income above me: $\mu_r(y) = E_F[Y \mid Y > y]$. Relative to this group I am worse off as $(\mu_r(y) - y)$. My net envy or subjective divergence is measured as the difference $v(y) = (\mu_r(y) - y) - (y - \mu_l(y))$. Then over the entire distribution of incomes, the aggregate net envy or divergence is

$$v = \int_0^\infty f(y) v(y) dy. \tag{2}$$

It remains to give a simple interpretive expression for the integral. This can be done by introducing the unit left and right entropic shifts of the original F, which are defined as follows. For densities:

$$f_L(y) = \xi_L(y) f(y); \qquad f_R(y) = \xi_R(y) f(y), \tag{3a}$$

where the shift factors (technically Radon–Nikodym derivatives) are given by

$$\xi_L(y) = -\ln F(y); \qquad \xi_R(y) = -\ln(1 - F(y)).$$

For the distribution functions:

$$\begin{aligned} F_L(y) &= F(y)(1 + \xi_L(y)); \\ 1 - F_R(y) &= (1 - F(y))(1 + \xi_R(y)). \end{aligned} \tag{3b}$$

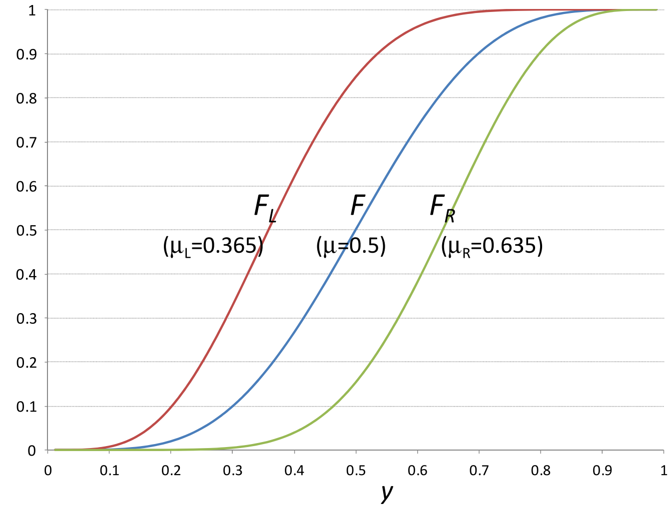

Fig. 1. The effects of left and right entropic shifts.

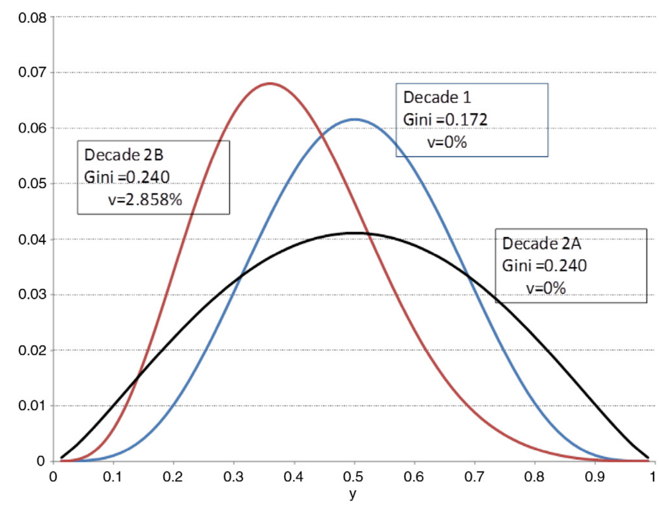

Fig. 2. Interpretive illustration. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The nature and properties of these entropic shifts, together with the parent concept of partition or locational entropy are explored in Bowden (2012) and have since found diverse applications to curve smoothing and edge correction, mark scaling, financial risk management, and opinion polling. Note that the entropy invoked is that of a dichotomous variable based on whether $Y \le y$ or $Y > y$; it is not the Shannon entropy of Y itself (which would be $E_F[y \ln(y)]$). The dichotomy refers to the 'look back' versus the 'look forward' aspect of subject y's income relativity.

Fig. 1 illustrates with an initially symmetric Beta(5,5) distribution together with its left and right unit entropic shifts. For a symmetric density the left and right unit shifts are anti-symmetric about the mean:

The following result gives a general relationship between expectations based on F_L and those based on the original F.

Lemma. *Let $g = g(y)$ be a measurable function such that $E_F[g(Y)] < \infty$, and let $\varphi(y)$ be the conditional expectation function defined by $\varphi(y) = E[g(Y) \mid Y \le y]$. Then*

$$E_{F_L}[g(y)] = E_F[\varphi(y)]. \tag{4}$$

Proof (*Integration by Parts*)**.** Thus an expectation with respect to the unit left shifted distribution can be regarded as a second layer