Economics Letters 134 (2015) 73–77

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Revisiting the optimal linear income tax with categorical transfers

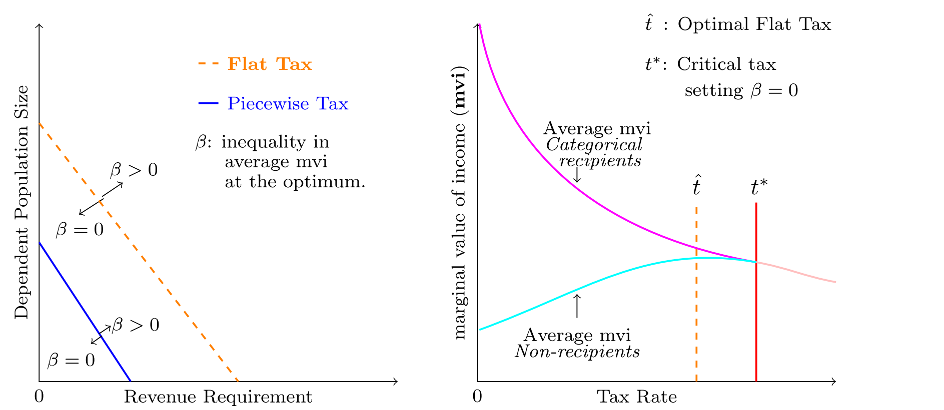
Sean Edward Slack*

School of Economics & Finance, University of St. Andrews, KY16 9AR, UK

HIGHLIGHTS

- Linear/piecewise-linear income tax finances categorical transfers to unable group.
- Inequality in average marginal value of income across unable and able groups.
- This inequality may persist at optimum: depends on unable group size/rev requirement.
- Optimal tax expressions can be written more generally to allow for this.
- Numerical results provide examples where this arises under both types of tax system.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 7 March 2015
Received in revised form 18 June 2015
Accepted 19 June 2015
Available online 26 June 2015

JEL classification:
H2
H53
I38

Keywords:
Between-group inequality
Categorical benefit
Linear income tax
Social marginal value of income

ABSTRACT

When individuals differ in both productivity and some categorical attribute, optimal linear/piecewise-linear tax expressions are written to capture cases where it is suboptimal to eliminate inequality in the average social marginal value of income between categorical groups. Simulations provide examples.

1. Introduction

When individuals differ in both their productivity and some categorical dimension such as disability, a well-established result is that categorical transfers should be set so as to eliminate inequality in the average social marginal value of income (smvi) between categorical groups (Diamond and Sheshinski, 1995; Parsons, 1996). The linear income tax framework has played an important role in the analysis of categorical transfers: proponents of flat tax schedules cite their administrative simplicity and enhanced work incentives; whilst analytically a flat tax captures the equity–efficiency tradeoff of income taxation more tractably

* Tel.: +44 0 1334 468305.
E-mail address: ses34@st-andrews.ac.uk.

http://dx.doi.org/10.1016/j.econlet.2015.06.015

than nonlinear taxation (Atkinson, 1995; Paulus and Peichl, 2009).[1] The resulting optimal tax formulae are typically reported under the assumption that inequality in the average *net* smvi is indeed eliminated at the optimum (Viard, 2001). This assumption allows the optimal tax expression to be written as in the uni-dimensional model where individuals differ only in productivity: the numerator (equity considerations) is the negative of the covariance between earnings and the *net* smvi; whilst the denominator (efficiency considerations) captures the compensated labour supply response to a change in the net wage rate.

However, it is not immediately clear that this between-group inequality will always be eliminated at the optimum. Indeed, there may be cases where it is suboptimal to do so: if a sufficiently large fraction of the population are dependent on categorical transfers for consumption then the level of taxation required to equate the average net smvi of dependent and non-dependent groups may be too harmful to the latter group. This will also depend on the size of any revenue requirement for spending outside welfare.

Moreover, this is likely to hold beyond a simple flat tax framework. For example, progressive piecewise linear tax systems provide the government with additional tools to redistribute *within* categorical groups; but if shifting some of the tax burden away from lower earners in an able group: (i) pushes the average net smvi of that group further below that of a dependent group; and/or (ii) lowers tax revenue relative to the flat tax case, this may limit further the cases where it is optimal to eliminate between-group inequality.

This paper addresses this issue in both linear and piecewise linear income tax frameworks. It demonstrates that the optimal tax expressions can be written more generally to allow for cases where the average net smvi of categorical groups are not equated at the optimum. In these cases welfare provision is purely categorical, such that no universal benefit is provided. Alternatively, if between-group inequality is eliminated *and* there are resources left over a universal benefit is also provided. This reflects somewhat the ordering of priorities in real-world welfare systems: whilst most systems feature dimensions of both categorical and universal support, the former plays the prominent role. Extensive numerical simulations provide examples where between-group inequality is not eliminated at the optimum. Further, they indicate that it is more likely to arise under a progressive piecewise system for the reasons outlined above.

2. The model

2.1. Background

Individual preferences over consumption, $x \geq 0$, and leisure, $l \in [0, 1]$, are represented by the utility function $u(x, l)$. The standard assumptions apply: u is continuous; differentiable; increasing in both arguments ($u_x > 0, u_l > 0$) and concave ($u_{xx} < 0, u_{ll} < 0, u_{xx}u_{ll} - u_{xl}^2 > 0$); with both goods normal ($u_l u_{xx} - u_x u_{xl} < 0$).

For an individual with net wage $\omega \geq 0$ and unearned income $M \geq 0$, optimal labour supply (H^*) and the resulting indirect utility function (v) are defined by:

$$H^*(\omega, M) \equiv \arg\max_{H \in (0,1)} u(\omega H + M, 1 - H),$$

$$v(\omega, M) \equiv u(\omega H^* + M, 1 - H^*).$$

Let $\bar{\omega}(M) = u_l(M, 1)/u_x(M, 1)$ be the reservation wage satisfying: $H^* = 0 \; \forall \omega \leq \bar{\omega}$ and $H^* > 0 \; \forall \omega > \bar{\omega}$; where $\bar{\omega}' > 0$. It follows that $\forall \; \omega < \bar{\omega}$: $v(\omega, M) = u(M, 1)$ and thus $v_M(\omega, M) = u_x(M, 1)$. Contrastingly, Roy's identity ($v_\omega = v_M H^*$) and the normality of leisure ($H_M^* < 0$) imply that $\forall \; \omega > \bar{\omega}$: $v_{\omega M} = v_{MM}H^* + v_M H_M^* < 0$. So for $\omega > \bar{\omega}$ the marginal indirect utility of unearned income is strictly decreasing in the net wage rate.

2.2. The tax-benefit system

Consider a population of size 1, where a fraction $\theta \in (0, 1)$ of individuals face a zero quantity constraint on labour supply and are thus unable to work. Absent any form of state financial provision these individuals would have zero income to consume. The remaining $(1 - \theta)$ individuals are able to work but differ in their underlying productivity $n \geq 0$, where n is distributed with density function $f(n)$ and associated distribution function $F(n)$.

The government operates a tax-benefit system comprising: (i) a constant marginal income tax rate $t \in (0, 1)$; (ii) a tax-free universal benefit $B \geq 0$ received unconditionally by all individuals in society; and (iii) a tax-free categorical benefit $C \geq 0$ that is perfectly targeted at unable individuals.

Let $y(n, 1 - t, M) \equiv nH^*[n(1 - t), M]$ be the gross earnings of a productivity n individual; whilst $\bar{y}(1 - t, M) \equiv \int yf(n)dn$ is the average gross earnings of able individuals.

Under a strictly utilitarian criterion, social welfare is:

$$W(t, B, C; \theta) = \theta u(B + C, 1) + (1 - \theta) \int v[n(1 - t), B]f(n)dn. \tag{1}$$

The government's problem is thus described by:

$$\begin{aligned} &\max_{t,B,C} W(t, B, C; \theta) \\ \text{s.t.} \quad & B + \theta C = (1 - \theta)t \cdot \bar{y}(t, B) - R, \\ & t \in (0, 1), \; B \geq 0, \; C \geq 0 \end{aligned} \tag{2}$$

where $R \geq 0$ is an exogenous revenue requirement.

To discuss the results which follow, let the net smvi of a productivity n individual be (Viard, 2001; Atkinson, 1995):

$$s(n, t, M, \lambda) = \begin{cases} u_x(M, 1) & : n \leq \bar{n}(t, M) \\ v_M[n(1 - t), M] + \lambda t y_M(n, 1 - t, M) & : n > \bar{n}(t, M) \end{cases} \tag{3}$$

where $\bar{n} \equiv \bar{\omega}/(1 - t)$ and λ is the shadow price of public expenditure. For working individuals s captures – in welfare units – the fact that an increase in unearned income induces a worker to reduce their labour supply and, consequently, lowers tax revenue.

Letting $\hat{t}$, $\hat{B}$ and $\hat{C}$ be the optima resulting from (2), we have:

Result 1. (i) *$\hat{C} > 0$ and $\hat{B} \geq 0$ satisfy:*

$$\bar{s}\left(\hat{t}, \hat{B}, \hat{\lambda}\right) \leq u_x\left(\hat{B} + \hat{C}, 1\right) = \hat{\lambda}; \quad \hat{B} \geq 0 \tag{4}$$

where the pair of inequalities holds with complementary slackness and $\hat{\lambda}$ is the shadow price of public expenditure at the optimum.

(ii) *For $\beta \equiv (\lambda - \bar{s})$ and $r = y/\bar{y}$; $\hat{t}$ is implicitly characterised by:*

$$\frac{\hat{t}}{1 - \hat{t}} = \begin{cases} \dfrac{\beta - \text{Cov}(r, s)}{\hat{\lambda} \int r\mathcal{E}^c f(n)dn} & : \beta > 0 \\ \dfrac{-\text{Cov}(y, s)}{\hat{\lambda} \int y\mathcal{E}^c f(n)dn} & : \beta = 0 \end{cases} \tag{5}$$

where $\mathcal{E}^c$ is the compensated elasticity of earnings with respect to the net of tax rate.

[1] Mirrlees (1971, p. 208) discusses the desirability of approximately linear tax schedules.