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A non-perpetual shirking model

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HIGHLIGHTS

- We derive a finite-horizon version of the Shapiro–Stiglitz shirking model of unemployment.
- Workers' behavior may change as they approach the end of an employment contract.
- Our model predicts that wages should be rising in age for an unchanged rate of unemployment.

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ABSTRACT

We provide a finite-horizon counterpart to the Shapiro and Stiglitz model of unemployment to show how workers' effort falls as they approach the end of an employment spell. The model provides a reason for wages rising more rapidly than productivity.

1. Introduction

Every employment contract has a time dimension. There are workers on temporary contracts; workers who have been given an advance notice of dismissal know that their days on the job are numbered; and even workers who have safe permanent jobs realize that they will eventually retire. In this paper we extend and generalize the well-known model of wage setting by Shapiro and Stiglitz (S–S) (1984) to show how a worker's propensity to shirk his duties varies from the beginning to the end of an employment contract.

2. A non-perpetual model

We model a worker's effort decision when he has finite horizons leaving the infinite horizon case described in the S–S paper as a special case. There are three states of intertemporal utilities in the S–S model for workers with transitory probabilities to alternative states. These are the values of being employed, V_E (when not shirking) and V_S (when shirking), and the value of being unemployed, V_U. Workers receive the wage w when employed and unemployment benefits b_u when unemployed. Effort is exerted when employed workers are not shirking their duties while no effort is exerted when workers shirk. Workers discount future utility at rate ρ, face a constant probability of job termination b during the contract period and the probability q of being fired if caught shirking.

We start with a representative state i

$$V_i = \int_t^{\infty} u_i(s)\, e^{-\rho(s-t)} ds, \tag{1}$$

with transitory probability p_{ij} of moving to the alternative state V_j, where $u_i(s)$ is the immediate utility at time s for the state i. We can now introduce finite horizons by dividing the inter-temporal integral V_i into the periods of $t \leq time \leq T$ and $T \leq time \leq \infty$

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where T denotes the time remaining until the end of horizon:

$$V_i = \int_t^{\infty} u_i(s)\, e^{-\rho(s-t)} ds = \int_t^{T} u_i(s)\, e^{-\rho(s-t)} ds + \int_T^{\infty} u_i(s)\, e^{-\rho(s-t)} ds. \quad (2)$$

The integral $\int_T^{\infty} u_i(s)\, e^{-\rho(s-t)} ds$ for time period $T \leq time \leq \infty$ can be rewritten as follows:

$$\int_T^{\infty} u_i(s)\, e^{-\rho(s-t)} ds = e^{-\rho(T-t)} \int_T^{\infty} u_i(s)\, e^{-\rho(s-T)} ds. \quad (3)$$

Therefore, we need to discount the integral by the factor $e^{-p_{ij}(T-t)}$ if we would like to replace T with t since over the time period from t to T, the integral $\int_T^{\infty} u_i(s)\, e^{-\rho(s-T)} ds$ depreciates at the rate of p_{ij}:

$$e^{-\rho(T-t)} \int_T^{\infty} u_i(s)\, e^{-\rho(s-T)} ds = e^{-(\rho+p_{ij})(T-t)} \int_t^{\infty} u_i(s)\, e^{-\rho(s-t)} ds. \quad (4)$$

Eq. (2) can now be rewritten as

$$V_i = \int_t^{T} u_i(s)\, e^{-\rho(s-t)} ds + e^{-(\rho+p_{ij})(T-t)} \times \int_t^{\infty} u_i(s)\, e^{-\rho(s-t)} ds = V_i^T + e^{-(\rho+p_{ij})(T-t)} V_i, \quad (5)$$

where $V_i^T = \int_t^T u_i(s)\, e^{-\rho(s-t)} ds$. Rearranging gives

$$V_i = \frac{V_i^T}{1 - e^{-(\rho+p_{ij})(T-t)}}. \quad (6)$$

Eq. (6) shows the relationship between the perpetual and non-perpetual intertemporal integrals for the state i. One can then apply Eq. (6) to three states: V_E, V_S, and V_U, with corresponding transitory probabilities: $p_{EU} = b$, $p_{SU} = b + q$, and $p_{UE} = a$, where $V_E^T = \int_t^T (w - \bar{e})\, e^{-\rho(s-t)} ds$ is the non-perpetual integral for the value of being a non-shirking employed worker who faces the probability b of moving to the unemployed state, $\bar{e}$ is the disutility of effort, $V_S^T = \int_t^T w e^{-\rho(s-t)} ds$ is the non-perpetual integral for the value of being a shirking worker who faces the probability $b+q$ of moving to the unemployment state and $V_U^T = \int_t^T b_u e^{-\rho(s-t)} ds$ is the non-perpetual integral for an unemployed worker who becomes employed with probability a, which denotes the probability of finding jobs.

We can derive the following three asset pricing equations by substituting Eq. (6) into the Bellman equations of the perpetual case of the S–S model;

$$\rho V_E^T = (w - \bar{e})\left(1 - e^{-(\rho+b)(T-t)}\right) + b\left(V_U^T \frac{1 - e^{-(\rho+b)(T-t)}}{1 - e^{-(\rho+a)(T-t)}} - V_E^T\right), \quad (7)$$

$$\rho V_S^T = w\left(1 - e^{-(\rho+b+q)(T-t)}\right) + (b+q)\left(V_U^T \frac{1 - e^{-(\rho+b+q)(T-t)}}{1 - e^{-(\rho+a)(T-t)}} - V_S^T\right), \quad (8)$$

$$\rho V_U^T = b_u\left(1 - e^{-(\rho+a)(T-t)}\right) + a\left(V_E^T \frac{1 - e^{-(\rho+a)(T-t)}}{1 - e^{-(\rho+b)(T-t)}} - V_U^T\right). \quad (9)$$

Using the no-shirking condition such that $V_E^T = V_S^T$ for Eq. (8) gives

$$\rho V_E^T = w\left(1 - e^{-(\rho+b+q)(T-t)}\right) + (b+q)\left(V_U^T \frac{1 - e^{-(\rho+b+q)(T-t)}}{1 - e^{-(\rho+a)(T-t)}} - V_E^T\right). \quad (10)$$

There are three unknown variables, V_E^T, V_U^T, w, for (7), (9) and (10). Rearranging those three equations gives

$$(\rho+b) V_E^T - b\left(\frac{1 - e^{-(\rho+b)(T-t)}}{1 - e^{-(\rho+a)(T-t)}}\right) V_U^T - w\left(1 - e^{-(\rho+b)(T-t)}\right) = -\bar{e}\left(1 - e^{-(\rho+b)(T-t)}\right), \quad (11)$$

$$(\rho+b+q) V_E^T - (b+q)\left(\frac{1 - e^{-(\rho+b+q)(T-t)}}{1 - e^{-(\rho+a)(T-t)}}\right) V_U^T - w\left(1 - e^{-(\rho+b+q)(T-t)}\right) = 0, \quad (12)$$

$$a\left(\frac{1 - e^{-(\rho+a)(T-t)}}{1 - e^{-(\rho+b)(T-t)}}\right) V_E^T - (\rho+a) V_U^T = -b_u\left(1 - e^{-(\rho+a)(T-t)}\right). \quad (13)$$

Finally, using Cramer's rule gives the no-shirking condition for wages (see Appendix for details)

$$w = \frac{(1 - B/A)\left[\bar{e}a(b+q) - b_u b(\rho+b+q)\right] + (B/A)\, b_u \rho q + \bar{e}\rho(a+b+\rho+q)}{(1 - B/A)\left[(\rho+b)(\rho+a) + aq\right] + \rho q}, \quad (14)$$

where $A = \left(1 - e^{-(\rho+b)(T-t)}\right)$ and $B = \left(1 - e^{-(\rho+b+q)(T-t)}\right)$. Note that since $A < B$ we find that $(1 - B/A)$ is negative. The numerator of (14) falls faster than the denominator and the firm needs to pay wages that rise as the end of the contract period approaches. Because the effective discount rate for the shirking state is $\rho+b+q$ and higher than the effective discount rate for the non-shirking state $\rho + b$, shirking is less harmful to workers whose contract will expire soon.

For the perpetual case, we have $A = B$. Thus the no-shirking condition becomes

$$w = \frac{b_u \rho q + \bar{e}\rho(a+b+\rho+q)}{\rho q} = b_u + \bar{e} + (a+b+\rho)\frac{\bar{e}}{q}, \quad (15)$$

which is the original no-shirking condition of Shapiro and Stiglitz. Now denote the number of employed workers of age t by L_t. In steady state, the outflow from employment to unemployment equals bL_t and should equal to inflow of workers from unemployment to employment $a(N_t - L_t)$ where N_t is the number of workers of age t in the labor force.

$$bL_t = a(N_t - L_t). \quad (16)$$

Thus $a + b = bL_t (N_t - L_t)^{-1} + b = bN_t (N_t - L_t)^{-1} = b/u_t$ and we get $a = b(1 - u_t)/u_t$. Substituting back into (14) gives the no-shirking condition in equilibrium as a relationship between wages and unemployment.

$$w = \frac{(1 - B/A)\left[\bar{e}b((1-u_t)/u_t)(b+q) - b_u b(\rho+b+q)\right]}{(1 - B/A)\left[(\rho+b)(\rho + b((1-u_t)/u_t)) + b((1-u_t)/u_t)q\right] + \rho q} + \frac{(B/A)\, b_u \rho q + \bar{e}\rho(b/u_t + \rho + q)}{(1 - B/A)\left[(\rho+b)(\rho + b((1-u_t)/u_t)) + b((1-u_t)/u_t)q\right] + \rho q}. \quad (17)$$

It follows that each cohort of workers has a distinct wage curve – or no-shirking constraint – described by Eq. (17).

The non-shirking constraint is drawn in Fig. 1 as an upward-sloping non-shirking constraint for different age groups with benchmark values below the figure. There are only small differences between young and middle-aged workers. But the wage curves for older workers are substantially higher. It follows that the wage – or unemployment – needed to prevent a 40-year old worker from shirking his duties is not much higher than that needed to prevent a 20 year old worker from doing so but a significantly higher wage is needed to prevent a 50 year old worker from shirking than is the case of the 40 year old one.

As shown in Fig. 2 we find that the wage required to prevent shirking rises rapidly in the 44–48 years age group when unemployment is 10%, in the 48–52 years age group when unemployment is 20% and in the 52–56 group when unemployment is a staggering 40%.