Economics Letters 134 (2015) 107–110

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Testing change in volatility using panel data

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ARTICLE INFO

Article history:
Received 28 April 2015
Accepted 19 June 2015
Available online 29 June 2015

Keywords:
Panel data model
Change in volatility
CUSUM process

ABSTRACT

The focus of this paper is to test the possible changes in the volatility of panel data. The test statistic is derived from a likelihood argument and it is based on the CUSUM method. Asymptotic distribution is derived under the no change null hypothesis and the consistency of the test is also established. Monte Carlo simulation shows the effectiveness and improvement of the proposed procedure over some of the existing testing procedures.

1. Introduction

There has been a rapidly growing interest in testing structure stability of panel data model in statistics and econometrics. Feng et al. (2010) discuss the estimation of a single point in panel models via a Wald-type statistic and Baltagi et al. (2012) extend it to allow for nonstationary regressors and innovations. Bai (2010) used the least squares and the quasi-maximum likelihood method to estimate the time of change (t_0) assuming that a change has occurred. Follow Bai's (2010) model, Horváth and Hušková (2012) propose a CUSUM-based test for mean of panel data models. Li et al. (2015) extend their method to test the change in variance of panel data models.

This article considers testing for possible changes in the volatility (variance) parameter of panel data, in which there are N series (variables), and each series has T observations. Recently, Li et al. (2015) proposed using a CUSUM-based test for variance change with panel data. However, their test procedure may suffer power loss in situations when some series are subject to an increase in volatility, while other series are subject to a decrease in volatility. The increase and decrease in volatility may cancel each other and result in a loss of power of their testing method. In this paper we extend Li et al.'s (2015) method to test common breaks such that our modified testing method has good powers under general volatility changes. Section 2 describes the model and lists assumptions. In Section 3 we propose a new CUSUM-based test and derive asymptotic distribution under the null hypothesis of no change in volatility. We also establish the consistency of the test. In Section 4 we use Monte Carlo simulations to examine the finite sample performance of our proposed test.

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http://dx.doi.org/10.1016/j.econlet.2015.06.016

2. Models and assumptions

Consider a panel data model with cross section unit N and time periods T, let

$$Y_{i,t} = \mu_i + e_{i,t}, \quad i = 1, \dots, N, \; t = 1, \dots, T. \tag{1}$$

In this article, we use a model where the innovations form a linear process:

$$e_{i,t} = \sum_{l=0}^{\infty} c_{i,l}\varepsilon_{i,t-1}, \quad 1 \le i \le N, \; 1 \le t \le T.$$

Assumption A. 1. $E(\varepsilon_{i,0}) = 0, E(\varepsilon_{i,0}^2) = 1, E\,|\varepsilon_{i,0}|^8 < \infty$ and $\limsup_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} E\,|\varepsilon_{i,0}|^8 < \infty$.
2. The sequences $\{\varepsilon_{i,t}, -\infty < t < \infty\}$ are independent of each other.
3. For every i the variables $\{\varepsilon_{i,t}, -\infty < t < \infty\}$ are *i.i.d.*
4. $|c_{i,l}| \le c_0(l+1)^{-\alpha}$ for all $1 \le i \le N$, $0 \le l < \infty$, with some c_0 and $\alpha > 2.5$.
5. There is $\delta > 0$ such that $a_i^2 \ge \delta^2$ with $a_i = \sum_{l=0}^{\infty} c_{i,l}$ for all $1 \le i \le N$.

Since

$$\lim_{T\to\infty} \frac{1}{T} E\left(\sum_{t=1}^{T} e_{i,t}\right)^2 = \sigma_i^2, \quad 1 \le i \le N,$$

from Assumptions A1 and A5, it is easy to obtain that $a_i^2 = \sigma_i^2$ and

$$\sigma_i^2 \ge \delta^2 \quad \text{for all } 1 \le i \le N.$$

This set-up is very similar to Horváth and Hušková (2012) and Li et al. (2015).

This article is to test the following hypothesis:

H_0: the variance of panel i does not change for all $1 \le i \le N$ during the observation period,

H_1: the variance of panel i changes from σ_i^2 to σ_i^{2*} for all $1 \le i \le N$ at time t_0,

where $\sigma_i^{2*} = \sigma_i^2 + \delta_i$, t_0 is unknown, $t_0 = \lfloor Tx \rfloor$, $x \in [0, 1]$, $\lfloor \cdot \rfloor$ denotes the integer part. Most importantly, σ_i^{2*} can be larger or smaller than σ_i^2 for different i. Li et al. (2015) only consider changes in one direction either $\sigma_i^2 \ge \sigma_i^{2*}$ for all i, or $\sigma_i^2 \le \sigma_i^{2*}$ for all i.

3. Test statistic and asymptotic theory

The test statistic is defined as follows:

$$U_n = \sup_{0 \le x \le 1} |U_{N,T}(x)|, \tag{2}$$

where

$$U_{N,T}(x) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left[W_{T,i}^2(x) - \frac{\lfloor Tx \rfloor (T - \lfloor Tx \rfloor)}{T^2} \right], \quad 0 \le x \le 1, \tag{3}$$

and

$$W_{T,i}(x) = \frac{1}{\hat{\varphi}_i} \frac{1}{\sqrt{T}} \left(\sum_{t=1}^{\lfloor Tx \rfloor} \hat{e}_{i,t}^2 - \frac{\lfloor Tx \rfloor}{T} \sum_{t=1}^{T} \hat{e}_{i,t}^2 \right),$$

with

$$\hat{e}_{i,t} = Y_{i,t} - \bar{Y}_T(i), \quad \bar{Y}_T(i) = \frac{1}{T} \sum_{i=1}^{T} Y_{i,t}.$$

If for any i, the errors $e_{i,t}$, $1 \le t \le T$ are i.i.d.,

$$\hat{\varphi}_i^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{e}_{i,t}^4 - \left(\frac{1}{T} \sum_{t=1}^{T} \hat{e}_{i,t}^2 \right)^2, \quad i = 1, \ldots, N. \tag{4}$$

If independence cannot be assumed, a kernel estimator is used:

$$\hat{\varphi}_i^2 = \frac{1}{T} \sum_{t=1}^{T} \left(\hat{e}_{i,t}^2 - \frac{1}{T} \sum_{t=1}^{T} \hat{e}_{it}^2 \right)^2 + 2 \sum_{s=1}^{T-1} K\left(\frac{s}{h}\right) \hat{\gamma}_{T,s}(i), \tag{5}$$

where

$$\hat{\gamma}_{T,s}(i) = \frac{1}{T-s} \sum_{t=1}^{T-s} \left(\hat{e}_{i,t}^2 - \frac{1}{T} \sum_{t=1}^{T} \hat{e}_{it}^2 \right) \left(\hat{e}_{i,t+s}^2 - \frac{1}{T} \sum_{t=1}^{T} \hat{e}_{it}^2 \right).$$

The function K is the kernel in the definition of $\hat{\varphi}_i^2$ and $h = h(T)$ is the window. Throughout this article, the following conditions are assumed on the kernel estimator,

Assumption B. 1. $K(0) = 1$,

2. $K(u) = 0$ if $|u| > a$ and $K(u)$ is Lipschitz continuous on $[-a, a]$ with some $a > 0$,
3. K has υ bounded derivatives in a neighbourhood of 0 and the first $\upsilon - 1$ derivatives of K are 0 at 0, where $\upsilon \ge 1$ is an integer,
4. $h = h(T) \to \infty$ and $\frac{h}{T} \to 0$ as $T \to \infty$,
5. $\frac{Nh^2}{T^2} \to 0$ and $\frac{N^{1/2}}{h^\tau} \to 0$, where $\tau = \min(\upsilon, \alpha - 1)$.

And the change point (t_0) can be estimated as

$$\hat{t}_0 = \lfloor T * \operatorname*{argmax}_{0 \le x \le 1} U_{N,T}(x) \rfloor. \tag{6}$$

Table 1
Critical values of $\sup_{0 \le x \le 1} |U(x)|$.

0.1	0.05	0.01
0.8892	0.9897	1.2048

Table 2
Empirical sizes for U_n under AR(1) process.

N/T	$\phi = 0$	$\phi = 0.1$	$\phi = 0.3$	$\phi = 0.5$
50/50	0.0680	0.0580	0.0460	0.0375
50/100	0.0510	0.0460	0.0365	0.0390
100/100	0.0495	0.0470	0.0415	0.0415
100/200	0.0395	0.0465	0.0355	0.0590
200/200	0.0430	0.0345	0.0430	0.0740

Theorem 1. *If H_0, Assumption A holds, and $N/T \to 0$ as $\min(N, T) \to \infty$, then*

$$U_n \xrightarrow{\mathcal{D}[0,1]} \sup_{0 \le x \le 1} |G(x)|,$$

where $G(x)$ is a Gaussian process with $EG(x) = 0$ and $EG(x)G(y) = E\left[\left(B^0(x)^2 - x(1-x)\right)\left(B^0(y)^2 - y(1-y)\right)\right]$, $B^0(x)$ denotes a standard Brownian bridge, $\xrightarrow{\mathcal{D}[0,1]}$ denotes the weak convergence of stochastic process in the Skorokhod space $\mathcal{D}[0, 1]$.

Theorem 2. *If H_1, Assumption A holds, and*

$$\frac{T}{N^{1/2}} \sum_{i=1}^{N} \delta_i^2 \to \infty, \quad 1 \le i \le N,$$

as $\min(N, T) \to \infty$, then

$$U_n \xrightarrow{P} \infty.$$

4. Simulations

In this section we use Monte Carlo simulations to examine the finite sample performance of our proposed test. First, to obtain the critical values, we need to approximate $U(x)$ by simulations.

$$P\{ \sup_{0 \le x \le 1} |U(x)| > z_\alpha \} = \alpha.$$

The critical values are calculated by generating 10 000 times $\sup_{0 \le x \le 1} |U_{N,T}(x)|$ when $N = 500$ and $T = 1000$. The results for $\alpha = 0.1$, 0.05 and 0.01 are given in Table 1.

Next, we consider the case that $e_{i,t}$ is not independent, let

$$e_{i,t} = \phi e_{i,t-1} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim N(0, 1).$$

Here we use the same flat-top kernel as Horváth and Hušková (2012) to estimate the long-run variance of Eq. (5),

$$K(u) = \mathbf{1}\left(|u| \le \frac{1}{2}\right) + 2(1 - |u|)\mathbf{1}\left(\frac{1}{2} \le |u| \le 1\right),$$

where $\mathbf{1}(A)$ is an indicator function that equals to one if A holds, 0 otherwise. In Horváth and Hušková (2012), they tried several values for h and $h \in [2.5, 5]$ worked well. Tables 2–3 report the empirical sizes and powers of U_n in case of AR(1) process, which is calculated at nominal level $\alpha = 0.05$ with $h = 3$. The power of the test is considered very briefly. Under the alternative hypothesis, the distribution of ϵ changes from standard normal distribution to t-distribution with degree of freedom 5 at time t_0 which means the variance of ϵ increases from 1 to $\frac{5}{5-2}$, where $t_0 = T/3, T/2$. All simulations are based on 2000 replications.