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Fractional Frequency Flexible Fourier Form to approximate smooth breaks in unit root testing

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HIGHLIGHTS

- Fractional Frequency Flexible Fourier Form-DF-type of unit root test is proposed.
- The small sample properties of FFFFF-DF-type test are better than EL test.
- FFFFF-DF-type test improves the empirical testing performance.
- FFFFF-DF-type test prevents type two errors and over-filtration problems.

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ABSTRACT

In this study, a Fractional Frequency Flexible Fourier Form DF-type unit root test is proposed. The small sample properties of the proposed test are found to be better than that of the integer frequency counterpart.

1. Introduction

In the recent literature, multiple smooth breaks have been modeled by Flexible Fourier Transforms by Becker et al. (2006), Enders and Lee (2012a,b), and Rodrigues and Taylor (2012). The advantages of the Fourier approach include being able to capture the behavior of a deterministic function of unknown form even if the function itself is not periodic, working better than dummy variable methods irrespective of whether the breaks are instantaneous or smooth, and avoiding the problems of selecting the dates, number and form of breaks (Becker et al., 2006, Enders and Lee, 2012a,b; Rodrigues and Taylor, 2012). All these papers pointed out that the single frequency component of the Fourier Transforms should be used for structural break determination; otherwise, the over-filtration problem arises. However, none of them seems to consider the Fractional Frequency version of their test in their studies. The Fractional Frequency Flexible Fourier Form (FFFFF) is used in Becker et al. (2004) for the structural break test, namely *Trig*-test. They attempt to prove that their methodology is better than the conventionally used break tests. Moreover, in their study Becker et al. (2004) show that the best fitting frequency for the US inflation rate between the period 1947:1 to 2011:11 is $f^* = 1.178$, which is fractionally determined when testing for structural breaks. Therefore, by combining the methodologies of Becker et al. (2004) and Enders and Lee (2012b, Henceforth, EL test), this study aims to improve the unit root testing with Fourier Transforms.

2. Unit root test with FFFFF

The following Dickey–Fuller test is considered;

$$y_t = d(t) + \phi_1 y_{t-1} + \lambda t + \varepsilon_t \quad (1)$$

where ε_t is a stationary disturbance with variance σ^2, and $d(t)$ is a deterministic function of t. We also note that the initial value

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Table 1
Critical values for $\tau^{fr}_{DF\_C}$.

k	$T = 100$			$T = 200$			$T = 500$			$T = 1000$		
	%10	%5	%1	%10	%5	%1	%10	%5	%1	%10	%5	%1
1.1	−3.42	−3.74	−4.39	−3.39	−3.72	−4.33	−3.38	−3.70	−4.27	−3.38	−3.68	−4.26
1.2	−3.33	−3.67	−4.31	−3.32	−3.64	−4.26	−3.31	−3.63	−4.23	−3.30	−3.62	−4.21
1.3	−3.26	−3.62	−4.29	−3.25	−3.58	−4.20	−3.24	−3.56	−4.19	−3.23	−3.56	−4.17
1.4	−3.20	−3.55	−4.22	−3.19	−3.53	−4.17	−3.17	−3.51	−4.12	−3.17	−3.51	−4.09
1.5	−3.13	−3.48	−4.14	−3.13	−3.47	−4.10	−3.12	−3.45	−4.07	−3.11	−3.45	−4.07
1.6	−3.07	−3.42	−4.10	−3.06	−3.40	−4.06	−3.06	−3.41	−4.05	−3.05	−3.39	−4.01
1.7	−3.01	−3.37	−4.06	−3.00	−3.36	−4.01	−3.01	−3.35	−3.99	−2.99	−3.34	−3.98
1.8	−2.97	−3.34	−4.00	−2.97	−3.32	−3.97	−2.96	−3.30	−3.95	−2.96	−3.31	−3.93
1.9	−2.94	−3.30	−3.99	−2.93	−3.29	−3.96	−2.94	−3.29	−3.94	−2.93	−3.28	−3.92
Critical values of $F\left(\hat{k}^{fr}\right) = MaxF\left(k^{fr}\right)$												
	8.78	10.29	13.48	8.50	9.85	12.76	8.33	9.64	12.37	8.31	9.60	12.31

is assumed to be a fixed value, and ε_t is weakly dependent as in Enders and Lee (2012a,b). As pointed out by Enders and Lee (2012a,b), if the functional form of $d(t)$ is known, it is possible to estimate Eq. (1) and to test the null hypothesis of a unit root. Also when the form of $d(t)$ is unknown, any test for $\phi_1 = 1$ is difficult if $d(t)$ is miss-identified. Our test and Enders and Lee (2012a,b) tests are based on the fact that it is possible to approximate $d(t)$ employing the Fourier expansion:

$$d(t) = \alpha_0 + \alpha \sin\left(\frac{2\pi kt}{T}\right) + \beta_k \cos\left(\frac{2\pi kt}{T}\right) \quad (2)$$

where k indicates a particular frequency, and T is the number of observations. When there is no nonlinear trend all values of $\alpha_k = \beta_k = 0$, and this leads to a special case of the test, namely the DF test. There are a lot of reasons why it is inappropriate to utilize a large number of cumulative frequencies. As recommended in the literature, specific frequency $k = 1$ often leads to a good approximation to a model with structural change. Following these advice, we also use the single frequency and neglect n which indicates the number of the frequency used in our testing procedure. Moreover, we use the fractional frequency instead of integer ones as stated in the introduction. Therefore, this fractional frequency methodology enables us to set an appropriate nonlinear trend into the unit root testing procedure.

For selecting the best fitting fractional single frequency, we can also follow Davies (1987), which uses a completely data driven method. The grid search method works as follows: run a regression using Eq. (2) by using the single frequency between the intervals $0.1 \leq k^{fr} \leq k^{fr}_{\max}$, where $k_{\max} = 2$ as recommended in the aforementioned references. However, for fractional frequencies, we select $k = 0.1$ as increments of the selected frequencies. Finally, we obtain the $k = \hat{k}^f$ that minimizes the SSR. Besides, $F\left(\hat{k}^f\right) = MaxF\left(k^f\right)$ test statistics are also obtained following Enders and Lee (2012a,b). The testing regression is:

$$\Delta y_t = \rho y_{t-1} + c_1 + c_2 t + c_3 \sin\left(\frac{2\pi k^{fr} t}{T}\right) + c_4 \cos\left(\frac{2\pi k^{fr} t}{T}\right) + e_t. \quad (3)$$

The obtained critical values in Table 1 are for only fractional frequency values since the integer ones are tabulated in Enders and Lee (2012b).[1]

The proposed FFFFF type DF test has the same asymptotic properties with Enders and Lee (2012b). Enders and Lee (2012b) also pointed out that the asymptotic properties of the DF version of the test are not different from those of the LM version. Similar with the integer frequencies, critical values for the null hypothesis of a unit root will depend only on the frequency k and the sample size T where coefficients of the Fourier function and any other deterministic terms do not affect the asymptotic distribution (see Table 2).

3. Finite sample performances

For the small sample experiments we design a similar Monte-Carlo experiment with Enders and Lee (2012b), however with a slight difference. Since we compare the power of the integer frequency test in a fractional frequency setting, we use $k^{fr} = 1.3$, 1.5 for data generating process (DGP). The following Table 3 gives the results of the power analysis of the test statistics.[2]

As it can be readily seen from Table 3, FFFFF unit test offers a better power than the integer ones.[3] The recommended frequency selection and F-test of nonlinear trend detection which are given in Enders and Lee (2012a,b) are for integer frequencies. Therefore, if the true or optimal frequency is a fractional frequency, we will see a power loss in the empirical testing process. The frequencies are given as 1.3 and 1.5 for the power DGP's, respectively. It is possible that the recommended data based frequency selection method selects the $k = 1$ frequency for these nonlinear trends. Therefore, the FFFFF test is approximately 20% better than the DF version of the EL test with respect to the power analysis carried out for $T = 100$ and contributes more to the higher values of the T dimension. If the true DGP is obtained by taking $k^{fr} = 1.5$, then performing an EL test with $k = 1$ will result with the highest power loss. On the other hand, for $kfr = 1.5$ or greater than this frequency, the data driven method may select the frequency as $k = 2.0$. Therefore, the EL test creates an over-filtration problem since it uses integer frequencies. Thus, using fractional frequencies may lead us to obtain the true or optimal nonlinear trends for modeling structural breaks.[4] Enders and Lee (2012a) state that improperly modeling the break can be as problematic as ignoring the break altogether. Therefore, integer value frequency may also be an improper way of modeling the break, which has already been extensively investigated in Enders and Lee (2012a).

[1] The critical values for and $k = \{0.9, \ldots, 0.00001\}$ are approximately −4.50, −3.90, and −3.60 for %10, %5, and %1, respectively. The critical values for and $k \cong \{e-8, \ldots, e-47\}$ are approximately −4.05, −3.45, and −3.15 for %1, %5, and %10, respectively. Finally, for $k < e - 48$ the critical values converge to the critical values of DF test −3.517, −2.898, and −2.584.

[2] We have skipped the size analysis since we have obtained the similar results with Enders and Lee (2012b) in order to save space.

[3] For $T = 500$ all fractional frequencies power value becomes 1.000. In order to save space, we have not tabulated them.

[4] We have obtained the similar results with intercept and trend version of the test; hence, in order to save more space, we skipped them. However, the power results are available upon request.