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The variance risk premium and fundamental uncertainty

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HIGHLIGHTS

- We suggest a new measure for the expected variance risk premium (VRP).
- The new VRP is based on a conditional variance forecast from a GARCH-MIDAS model.
- The long-term volatility component reflects the state of the macroeconomy.
- Our VRP has stronger predictive power for stock returns than conventional measures.
- Long-term volatility component captures the fundamental uncertainty driving the VRP.

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ABSTRACT

We propose a new measure of the expected variance risk premium that is based on a forecast of the conditional variance from a GARCH-MIDAS model. We find that the new measure has strong predictive ability for future US aggregate stock market returns and rationalize this result by showing that the new measure effectively isolates fundamental uncertainty as the factor that drives the variance risk premium.

1. Introduction

The findings in Bollerslev et al. (2009, 2012, 2014), Bekaert and Hoerova (2014) and others strongly suggest that the variance risk premium (VRP) predicts medium-term aggregate stock market returns. Economically, the predictive ability of the VRP can be rationalized by its close relation to economic uncertainty and aggregate risk aversion (see Bollerslev et al., 2009, 2011) or (Corradi et al., 2013).[2]

Formally, the expected VRP is defined as the difference between the ex-ante risk-neutral expectation of future stock market variation and the statistical expectation of the realized variance. While 'model-free implied volatilities' can be constructed from option prices, the expected realized variance has to be estimated. The most common approaches are either to assume that the realized variance follows a martingale difference or to estimate a heterogeneous autoregressive model for the realized variance (HAR-RV).

We follow a different approach by modeling the conditional variance of daily stock returns as a GARCH-MIDAS process. In

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2 Using a stylized self-contained general equilibrium model, Bollerslev et al. (2009) show that the equity risk premium can be decomposed into two terms. While the first term describes the classical risk-return trade-off, the second one suggests a positive relation between expected returns and the volatility of consumption growth volatility (vol-of-vol). The predictive ability of the VRP then follows from the observation that the VRP is proportional to the time-varying vol-of-vol.

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this setting, the conditional variance is decomposed into a short-term GARCH component and a long-term component that is driven by macroeconomic explanatory variables. As discussed in Conrad and Loch (forthcoming), we think of the long-term component as 'the part' of the conditional variance of stock market returns that is driven by "uncertainty about the variability of economic prospects" (Bollerslev et al., 2013, p. 417).

Our contribution to the literature on the VRP is twofold. First, we suggest a new proxy for the expected VRP that is based on the difference between the option-implied variance and the variance forecast from the GARCH-MIDAS model. We then show that the proposed measure has considerably stronger predictive power for stock returns than conventional measures of the VRP. Second, we rationalize the strong predictive power of our new measure by showing that it effectively isolates the long-term volatility component as the factor that determines the VRP.

2. A new variance risk premium measure

2.1. The GARCH-MIDAS model

The GARCH-MIDAS model specifies the conditional variance of daily returns as the product of a short-term GARCH component that captures day-to-day fluctuations in volatility and a long-term component that is entirely driven by low-frequency (monthly) macroeconomic variables. The long-term component fluctuates at the monthly frequency only and can be considered as representing economic or fundamental uncertainty. Following Conrad and Loch (forthcoming), we denote daily returns by $r_{i,t}$, where t refers to a certain month and $i = 1, \ldots, N^{(t)}$ to the i'th day within that month. We then assume that

$$r_{i,t} = \mu + \sqrt{g_{i,t}\tau_t} Z_{i,t}, \quad (1)$$

where $Z_{i,t}$ is IID with mean zero and variance one. $g_{i,t}$ and τ_t represent the short- and long-term conditional variances, which are measurable with respect to the information set given at day $i-1$ of month t. The short-term component follows a mean-reverting asymmetric unit variance GARCH process

$$g_{i,t} = (1 - \alpha - \beta - \gamma/2) + \left(\alpha + \gamma \cdot \mathbb{1}_{\{r_{i-1,t} - \mu < 0\}}\right) \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{i-1,t}, \quad (2)$$

with $\alpha > 0$, $\beta > 0$ and $\alpha + \beta + \gamma/2 < 1$. The long-term component is driven by lagged values of an explanatory variable X_t:

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k}, \quad (3)$$

where the behavior of the MIDAS weights $\varphi_k(\omega_1, \omega_2)$ is parsimoniously determined using a flexible Beta weighting scheme. For a more detailed discussion, see Engle et al. (2013) or Conrad and Loch (forthcoming).

At the last day of each month t, we use the GARCH-MIDAS (GM) model to construct out-of-sample forecasts for the realized variance during the following month, RV_{t+1}. Note that next month's long-term volatility, τ_{t+1}, is predetermined with respect to macro realizations up to month t. Then, the realized variance prediction is given by

$$\widehat{RV}^{GM}_{t+1} = \mathbf{E}_t \left[\sum_{i=1}^{N^{(t+1)}} g_{i,t+1}\tau_{t+1} Z^2_{i,t+1} \right] = \tilde{g}_{t+1}\tau_{t+1}, \quad (4)$$

where $\tilde{g}_{t+1} = \left(N^{(t+1)} + (g_{1,t+1} - 1) \frac{1-(\alpha+\beta+\gamma/2)^{N^{(t+1)}}}{1-\alpha-\beta-\gamma/2} \right)$. For a given value of the monthly short-term variance, $\tilde{g}_{t+1}$, a high (low) value of fundamental uncertainty, τ_{t+1}, will upscale (downscale) the forecast of the expected monthly realized variance. In this sense, τ_{t+1} is similar to the vol-of-vol factor in the model of Bollerslev et al. (2009).

2.2. Constructing the VRP

We define the monthly expected VRP as $IV_t - \mathbf{E}_t[RV_{t+1}]$, where IV_t is the risk-neutral expected variation during month $t + 1$ and $\mathbf{E}_t[RV_{t+1}]$ is the expected (under the physical measure) realized variation for that period. We build on the approximation of the expected VRP in Bollerslev et al. (2009) and measure IV_t by the end-of-month t value of the squared VIX and, assuming that RV_t follows a martingale difference sequence, replace $\mathbf{E}_t[RV_{t+1}]$ by RV_t. The VRP is thus given by

$$VRP_t = VIX^2_t - RV_t. \quad (5)$$

This measure is both directly observable and model-free. However, as discussed in Bekaert and Hoerova (2014), the assumption that RV_t follows a martingale difference sequence may be inappropriate. As a new measure, we propose to base the expected VRP on the conditional variance forecast from the GARCH-MIDAS model, $\widehat{RV}^{GM}_{t+1}$. This forecast explicitly takes into account the macroeconomic uncertainty via the long-term component:

$$VRP^{GM}_t = VIX^2_t - \widehat{RV}^{GM}_{t+1}. \quad (6)$$

3. Data

We use daily continuously compounded returns, $r_{i,t}$, for the S&P 500 and monthly US macroeconomic data from 1970 to 2011. We include industrial production growth (annualized month-to-month percentage change), the new orders index of the Institute for Supply Management (levels) and the Chicago Fed National Activity Index (NAI).[3] Annualized monthly excess returns are calculated as $r^{ex}_t = 12 \cdot (r_t - r_{f,t})$, where $r_t = \sum_{i=1}^{N^{(t)}} r_{i,t}$ and $r_{f,t}$ denotes the one-month T-bill rate. For the 2000 to 2011 period, we employ observations for the 'new' VIX and daily realized volatilities, $RV_{i,t}$, based on 5-minute intra-day returns obtained from the website of the Oxford-Man Institute of Quantitative Finance. Monthly realized variances are constructed as $RV_t = \sum_{i=1}^{N^{(t)}} RV_{i,t}$. Otherwise, all data are obtained from the FRED database at the Federal Reserve Bank of St. Louis.

4. Empirical results

4.1. VRP estimation

We estimate the GARCH-MIDAS models for the 1973 to 1999 period. Following Conrad and Loch (forthcoming), we include three MIDAS lag years of the macro variables and use a restricted ($\omega_1 = 1$, i.e. strictly decreasing) Beta weighting scheme. The estimation results presented in Table 1 basically replicate the findings in Conrad and Loch (forthcoming) but for a briefer sample. Specifically, for all variables the estimate of θ is highly significant and negative, thus confirming the counter-cyclical behavior of long-term volatility. Periods of economic growth above trend (e.g. measured by positive NAI realizations) are associated with a decline in long-term

[3] The NAI is a weighted average of 85 monthly national economic indicators. Positive realizations indicate growth above trend, while negative realizations indicate growth below trend. Industrial production and new orders are among the indicators considered.