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An empirical input allocation model for the cost minimizing multiproduct firm

Manhong Zhu, Gulcan Onel, James L. Seale Jr. *

Food and Resource Economics Department, University of Florida, United States

HIGHLIGHTS

- We focus on the multiproduct firm that minimizes cost before maximizing profit.
- A linear parameterization of the theoretical input allocation model is derived.
- From the linear parameterization, we formulate an empirical input allocation model.
- In general form, the empirical model allows for joint production.
- Input–output separability and input independence can be tested empirically.

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ABSTRACT

Laitinen and Theil (1978) derive a theoretical input allocation model for a multiproduct firm that first minimizes cost and second maximizes profit. However, its empirical counterpart is not available. We linearize the model and derive a general empirically estimable model.

1. Introduction

Laitinen and Theil (1978) derive a theoretical cost-based input allocation model for the multiproduct firm that faces a competitive input market and minimizes cost given input prices and output quantities with the objective of profit maximization. However, the theoretical cost-based input allocation model, in its general form, has proven too difficult to estimate empirically because of the complexity of the term involving output changes. In response, two empirical strategies have emerged. The first is to estimate a restricted cost-based input allocation model. For example, Clements and Theil (1978) and Theil and Clements (1978) simplify the input allocation model by imposing the restrictions of input–output separability and input independence. The former restriction forces the term involving output changes to zero, but the resulting input allocation decision is independent of changes in individual outputs or output prices, and it is identical to that of the single-product firm (Theil, 1977). The latter restriction simplifies the input-price term of the input allocation model to only involve the own-input price deflated by the Frisch input-price index. In this case, no input is a specific substitute or complement of any other input.

The second strategy is to estimate input demand and output supply systems (Laitinen, 1980; Rossi, 1984). However, this approach has inherent weaknesses. One is that the two systems

* Correspondence to: P.O. Box 110240 MCCB, Gainesville, FL 32611-0240, United States. Tel.: +1 352 256 5917; fax: +1 352 392 3646.

E-mail addresses: zhumanhong@ufl.edu (M. Zhu), gulcan.onel@ufl.edu (G. Onel), jseale@ufl.edu (J.L. Seale Jr.).

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cannot be estimated simultaneously. Also, the theoretical adding-up conditions of the input demand equations are violated in empirical application. Laitinen (1980) and Livanis and Moss (2006) recommend adding a residual correction term to the input demand equations that is the difference between the Divisia input index and the Divisia output index.[1] However, this approach may lead to erroneous economic interpretations because the added residual correction term may be attributed to changes in productivity instead of the "inexactness".

Recently, Seale et al. (2014) derived an empirical input allocation model for Laitinen's (1980) revenue-based multiproduct firm. In this paper, we derive the linear form of Laitinen and Theil's (1978) cost-based input allocation model and, based on it, an empirical input allocation model for the multiproduct firm. The resulting empirical model is general in that the input allocation decisions are a function of the Divisia input volume index, changes in individual input prices, and changes in individual output prices. Theoretical adding-up conditions hold automatically, and the theoretical restrictions of homogeneity, input–output separability, and input independence may be imposed and statistically tested.

2. Input allocation model

The input allocation equation for the ith input of the cost-minimizing multiproduct firm is (Laitinen and Theil, 1978)

$$f_i d\log q_i = \theta_i d(\log Q) + \gamma \sum_r g_r(\theta_i^r - \theta_i) d(\log z_r) - \psi \sum_j \theta_{ij} d\left(\log \frac{w_j}{W'}\right) \quad (1)$$

where $f_i = w_i q_i / C$ is the share of the ith input in total cost, $C = \sum_i w_i q_i$, where w_i and q_i are the price and quantity, respectively, of input $i(=1, \ldots, n)$; $d(\log Q) = \sum_i f_i d(\log q_i)$ is the Divisia input volume index; $g_r = p_r z_r / \sum_r p_r z_r = p_r z_r / R$ is the revenue share of output $r(=1, \ldots, m)$ where p_r and z_r are the price and quantity, respectively, of output r; $\theta_i^r = \frac{\partial(w_i q_i)/\partial z_r}{\partial C/\partial z_r}$ is the share of input i in the marginal cost of output r; $\theta_i = \sum_r g_r \theta_i^r = \sum_j \theta_{ij}$ is the share of input i in total cost; $\theta_{ij}'s$ are normalized price coefficients of the input allocation system and are elements of the symmetric positive definite matrix $\boldsymbol{\Theta} = [\theta_{ij}]$ such that $\sum_i \sum_j \theta_{ij} = \sum_i \theta_i = 1$ (Laitinen and Theil, 1978); $d(\log W') = \sum_i \theta_i d \log(w_i)$ is the Frisch input-price index; and $\gamma = R/C$ is the revenue–cost ratio under profit maximization. Finally, ψ is the inverse of the Divisia elasticity of total marginal cost and is defined as $\iota_n' \mathbf{F}(\mathbf{F} - \gamma \mathbf{H})^{-1} \mathbf{F} \iota_n$ where ι_n is an $n \times 1$ vector of ones, $\mathbf{F}$ is an $n \times n$ diagonal matrix of factor shares, $[f_i]$, and $\mathbf{H} = \left[\partial^2 h / \partial \log \mathbf{q} \partial \log \mathbf{q}'\right]_{n \times n}$ is the matrix of second derivatives of the production function (i.e., $h(\mathbf{q}, \mathbf{z}) = 0$) with respect to $\log \mathbf{q}$, an $n \times 1$ vector containing the $\log q_i$'s.

If one assumes input–output separability, $\theta_i^r = \theta_i \forall r$, and the output term, $\gamma \sum_r g_r(\theta_i^r - \theta_i) d(\log z_r)$, in Eq. (1) goes to zero restricting input allocation decisions to be a function of input prices and total inputs, but not outputs or output prices. If input independence is assumed, $\theta_{ij} = 0$ for $i \neq j$ and $\theta_{ij} = \theta_i$ for $i = j$, and the input-price term in Eq. (1) simplifies to $-\psi \theta_i d(\log w_i / W')$.

3. A linear input allocation model

In this section, a linear form of Eq. (1) is derived. First, using the definition above for $d(\log W')$, decompose the input-price term of Eq. (1) into two terms,

$$-\psi \sum_j \theta_{ij} d(\log w_j) + \psi \theta_i \sum_j \theta_j d(\log w_j). \quad (2)$$

Collecting terms, this becomes

$$-\psi \sum_j (\theta_{ij} - \theta_i \theta_j) d(\log w_j) = \sum_j \pi_{ij} d\left(\log w_j\right) \quad (3)$$

where, as shown in the Appendix, π_{ij} has the following properties: adding-up, $\sum_i \pi_{ij} = 0$; homogeneity, $\sum_j \pi_{ij} = 0$; and symmetry, $\pi_{ij} = \pi_{ji} \, \forall i, j$.

The multiproduct firm's output supply equation for the rth product implied by profit maximization is (Laitinen and Theil, 1978)

$$g_r d(\log z_r) = \psi^* \sum_s \theta_{rs}^* d\left(\log \frac{p_s}{W'^s}\right) \quad (4)$$

where the change in the firm's supply of output r is a linear function of all output prices, each deflated by its own Frisch input-price index, $d(\log W'^s) = \sum_j \theta_j^s d(\log w_j)$. In addition, θ_{rs}^* is an element of $\boldsymbol{\Theta}^* = [\theta_{rs}^*]$, an $m \times m$ symmetric positive definite matrix that is normalized such that $\sum_r \sum_s \theta_{rs}^* = \sum_r \theta_r^* = 1$ (using $\sum_s \theta_{rs}^* = \theta_r^*$ and $\sum_r \theta_r^* = 1$ (Laitinen and Theil, 1978)). Further, ψ^* satisfies the condition that $\psi^* \geq \psi/(\gamma - \psi) > 0$, and it is the price elasticity of the supply of the firm's total output (Laitinen and Theil, 1978).

Substituting Eq. (4) into $\gamma \sum_r g_r(\theta_i^r - \theta_i) d(\log z_r)$ of Eq. (1) yields an output-price term and an input-price term,

$$\bar{\psi} \sum_r (\theta_i^r - \theta_i) \sum_s \theta_{rs}^* d(\log p_s) - \bar{\psi} \sum_r (\theta_i^r - \theta_i) \sum_s \theta_{rs}^* d(\log W'^s) \quad (5)$$

where $\bar{\psi} = \gamma \psi^*$. The output-price term of Eq. (5) can be written as

$$\bar{\psi} \left[\sum_s \sum_r \theta_i^r \theta_{rs}^* d(\log p_s) - \theta_i \sum_s \sum_r \theta_{rs}^* d(\log p_s) \right]. \quad (6)$$

Define $\bar{\theta}_{is} = \sum_r \theta_i^r \theta_{rs}^*$ as a normalized output-price coefficient such that $\sum_i \sum_s \bar{\theta}_{is} = 1$ with the properties, as shown in the Appendix, that $\sum_i \sum_s \bar{\theta}_{is} = \sum_i \bar{\theta}_i = 1$ and $\sum_i \bar{\theta}_{is} = \theta_s^*$. Accordingly, Eq. (6) can be simplified further to

$$\bar{\psi} \sum_s (\bar{\theta}_{is} - \theta_i \theta_s^*) d(\log p_s) = \sum_s \bar{\pi}_{is} d(\log p_s) \quad (7)$$

where $\bar{\pi}_{is}$ has the properties that $\sum_i \bar{\pi}_{is} = 0$ (adding-up) and $\sum_s \bar{\pi}_{is} = \bar{\psi}(\bar{\theta}_i - \theta_i)$. Accordingly, the homogeneity condition does not generally hold for $\bar{\pi}_{is}$. The dimension of the matrix $\bar{\boldsymbol{\pi}} = [\bar{\pi}_{is}]$ is $n \times m$, and therefore, in general, symmetry does not hold.

Using $d(\log W'^s) = \sum_j \theta_j^s d(\log w_j)$, the input-price term of Eq. (5) can be written as

$$-\bar{\psi} \sum_r \theta_i^r \sum_s \theta_{rs}^* \sum_j \theta_j^s d(\log w_j) + \bar{\psi} \sum_r \theta_i \sum_s \theta_{rs}^* \sum_j \theta_j^s d(\log w_j). \quad (8)$$

Define $\bar{\theta}_{rj} = \sum_s \theta_{rs}^* \theta_j^s$ so the term above simplifies to

$$-\bar{\psi} \sum_r \theta_i^r \sum_j \bar{\theta}_{rj} d(\log w_j) + \bar{\psi} \theta_i \sum_j \sum_r \bar{\theta}_{rj} d(\log w_j). \quad (9)$$

[1] Laitinen (1980, p. 113) defines the residual correction term as $E_t = DQ_t - \bar{\gamma}_t DZ_t$ where DQ and DZ are the Divisia input volume and Divisia output volume indexes, respectively.