Economics Letters 131 (2015) 1–4

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Existence of steady-state equilibria in matching models with search frictions

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HIGHLIGHTS

- We study two-sided matching models with search frictions.
- We develop a new approach to prove existence of steady-state equilibria.
- Two assumptions are shown to suffice for the existence of steady-state equilibria.
- Models with transferable and nontransferable utilities satisfy these assumptions.

ARTICLE INFO

Article history:
Received 10 December 2014
Received in revised form
11 March 2015
Accepted 19 March 2015
Available online 26 March 2015

JEL classification:
C62
C78
D83

Keywords:
Search
Matching
Bargaining
Steady-state equilibrium

ABSTRACT

We prove existence of steady-state equilibrium in a class of matching models with search frictions.

1. Introduction

This paper contributes to the literature on the pairwise matching of heterogeneous agents with search frictions. The basic structure of our model is as in Shimer and Smith (2000) or Smith (2006): There is a continuum of infinitely lived agents who are either matched or unmatched at any moment in time. Meetings between unmatched agents are generated by an exogenous search technology. Upon meeting, agents play a bargaining game, determining whether or not they become matched and, provided they do so, their payoffs within the match. The main concern of the literature studying such models (surveyed in Smith, 2011) is the characterization of matching patterns in steady-state equilibria. Here we focus on the question of existence of steady-state equilibria.

Previously, this question has been addressed using two distinct approaches. Shimer and Smith (2000) provide an existence argument applicable both to models with transferable and nontransferable utilities (see Smith, 2006), but requiring that – given the agents' decisions which matches to accept – there is a unique distribution of unmatched agents which maintains a steady state. To obtain this uniqueness property, Shimer and Smith (2000) impose the stringent assumption of a quadratic search technology.[1] In contrast, the approach developed in Manea (2014a) accommodates

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[1] While Nöldeke and Tröger (2009) refine the existence argument from Shimer and Smith (2000) to cover linear search technologies neither proof extends to general search technologies, see Manea (2014a) for further discussion.

http://dx.doi.org/10.1016/j.econlet.2015.03.023
0165-1765/

general search technologies, but requires the uniqueness of equilibrium payoffs in an auxiliary model in which the distribution of unmatched agents searching for a partner is taken as given. While Manea (2014a) establishes this uniqueness property for a model with transferable utility, it is evident from Adachi (2003) that, in general, this property fails with nontransferable utility.[2]

In the following we develop an approach to prove existence of steady-state equilibrium which dispenses with the uniqueness requirements in Shimer and Smith (2000) and Manea (2014a). This yields existence under minimal regularity conditions on the search technology, akin to the ones introduced in Manea (2014a), and the bargaining problem faced by the agents when deciding on a match, allowing for both transferable and nontransferable utilities.

2. Model

There is an exogenous measure $\theta_i > 0$ of players of type $i \in N = \{1, \dots, n\}$, or simply players i.[3] Players are risk neutral, infinitely lived, and discount payoffs at rate $r > 0$. Time is continuous. We consider steady states in which a measure $\mu_i > 0$ of players i is unmatched and a measure $\theta_i - \mu_i > 0$ is matched. The unmatched players search: each unmatched player i meets unmatched players j at Poisson rate $\rho_{ij}(\mu) \geq 0$, where $\mu = (\mu_1, \dots, \mu_n) \in \mathbb{R}^n_{++}$.

Assumption 1. There exists a continuous function $m : \mathbb{R}^n_+ \to \mathbb{R}^{n\times n}_+$, satisfying (i) $m_{ij}(\mu) = m_{ji}(\mu)$ for all $\mu \in \mathbb{R}^n_+$ and (ii) $m_{ij}(\mu) = 0$ whenever $\mu_i = 0$, such that

$$\rho_{ij}(\mu) = \frac{m_{ij}(\mu)}{\mu_i}, \quad \forall \mu \in \mathbb{R}^n_{++}. \tag{1}$$

Assumption 1 is similar to the assumption on meeting rates in Manea (2014a). From Eq. (1), $m_{ij}(\mu) = \rho_{ij}(\mu)\mu_i$. Hence, $m_{ij}(\mu)$ is the mass of players i who meet a player j per unit time, which – as stated in part (i) of Assumption 1 – should equal the mass $m_{ji}(\mu) = \rho_{ji}(\mu)\mu_j$ of players j who meet a player i per unit time. As the mass of meetings is assumed continuous in the distribution of unmatched types, part (ii) of Assumption 1 is the natural boundary condition that the mass of meetings involving players i approaches zero as μ_i vanishes (see Stevens, 2007).

When unmatched players i and j meet, they play a bargaining game determining whether or not they form a match – enter a relationship – and if they do, the flow payoffs that they obtain until their match dissolves. Each match is dissolved randomly at Poisson rate $\omega > 0$. Separated partners return to the pool of unmatched agents. If players do not agree to match, they instantaneously return to the pool of unmatched agents.

If a fraction $a_{ij} \in [0, 1]$ of meetings between unmatched agents i and j results in a match, then the outflow of players i from the pool of unmatched agents per unit time is $\sum_{j\in N} a_{ij}m_{ij}(\mu)$. The inflow of such players into the unmatched pool is given by the mass of matched players i multiplied with the exogenous rate at which the matches of such players are dissolved, i.e., $\omega(\theta_i - \mu_i)$. In a steady state, inflows and outflows must balance, delivering the *balance condition*

$$\omega\,(\theta_i - \mu_i) = \sum_{j\in N} a_{ij}m_{ij}(\mu), \quad \forall i \in N. \tag{2}$$

Denote by v_i the expected (continuation) value of an unmatched player i and by v_{ij} the expected value of a player i conditional on a meeting with a player j before the bargaining in the pair has commenced. The flow payoff of unmatched players is zero. Because a player i meets a player j at rate $\rho_{ij}(\mu)$ and such a meeting results in a capital gain of $g_{ij} = v_{ij} - v_i$, we have the *value condition*

$$r v_i = \sum_{j\in N} \rho_{ij}(\mu) g_{ij}, \quad \forall i \in N. \tag{3}$$

Agents are free to refuse to enter a relationship, so that $v_i \geq 0$ and $g_{ij} \geq 0$. In the following $v = (v_1, \dots, v_n) \in \mathbb{R}^n_+$ denotes the vector of continuation values, the matrix $g \in \mathbb{R}^{n\times n}_+$ collects the gains g_{ij}, and the set of feasible matching probabilities a_{ij} is

$$A = \{a \in [0, 1]^{n\times n} \mid a_{ij} = a_{ji},\ \forall (i, j) \in N \times N\}. \tag{4}$$

The matching probabilities a_{ij} and the gains g_{ij} are determined by an equilibrium in the bargaining game between players i and j. We treat the bargaining game as a "black box" by specifying a bargaining correspondence $E : \mathbb{R}^n_+ \rightrightarrows A \times \mathbb{R}^{n\times n}_+$, mapping vectors of continuation values v into outcomes (a, g). The interpretation is that (a, g) is an equilibrium outcome in the collection of bilateral bargaining games induced by a vector of continuation values v if and only if (a, g) satisfies the *bargaining condition*

$$(a, g) \in E(v). \tag{5}$$

Assumption 2. The bargaining correspondence $E : \mathbb{R}^n_+ \rightrightarrows A \times \mathbb{R}^{n\times n}_+$ is upper hemicontinuous with $E(v)$ non-empty, closed, and convex-valued for all $v \in \mathbb{R}^n_+$. Further, there exists $\bar{g} \in \mathbb{R}_+$ such that $E(v) \subset A \times [0, \bar{g}]^{n\times n}$ holds for all $v \in \mathbb{R}^n_+$.

Section 4 derives the bargaining correspondence for two common specifications of the bargaining problem. In both cases, Assumption 2 is satisfied.

Definition 1. A steady-state equilibrium is a tuple $(\mu, v, a, g) \in \mathbb{R}^n_{++} \times \mathbb{R}^n_+ \times A \times \mathbb{R}^{n\times n}_+$ satisfying the balance condition (2), the value condition (3), and the bargaining condition (5).

3. Result

Proposition 1. *A steady-state equilibrium exists if Assumptions 1 and 2 hold.*

The idea underlying the proof of Proposition 1 is as follows: We may rewrite the balance condition (2) as

$$\mu_i = \frac{\theta_i - \sum_{j\in N} a_{ij}m_{ij}(\mu)}{\omega}, \quad \forall i \in N \tag{6}$$

and the value condition (3) as

$$v_i = \frac{\sum_{j\in N} \rho_{ij}(\mu) g_{ij}}{r}, \quad \forall i \in N. \tag{7}$$

Together with the bargaining condition (5) the right sides of (6) and (7) define a mapping $(\mu, v, a, g) \longmapsto (\mu', v', a', g')$. Fixed points of this mapping coincide with steady-state equilibria. Assumptions 1 and 2 then ensure that the existence of steady-state equilibria can be inferred from Kakutani's fixed point theorem.

[2] For sufficiently low search frictions, equilibrium payoffs in Adachi's model are not uniquely determined whenever there are multiple stable matchings in the frictionless marriage market serving as a benchmark for his analysis. Note that there is nothing pathological about multiplicity of stable matchings in frictionless marriage markets with nontransferable utility (cf. Roth and Sotomayor, 1990, Example 2.17).

[3] We follow Manea (2014a) in considering a finite type space rather than a continuum of types, thus sidestepping technicalities – but not, as discussed in Smith (2011), the substantive issues – in Shimer and Smith (2000). In a similar vein, we follow most of the literature in suppressing the measure theoretic considerations discussed and resolved in Manea (2014b).