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Gambling in contests with heterogeneous loss constraints

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HIGHLIGHTS

- I study an asymmetric stochastic contest model.
- Players differ in their ability to make debt.
- The unique equilibrium outcome and payoffs are characterized in closed form.
- A higher debt level of a player changes the bankruptcy risk of both players.
- The similarity of the equilibrium to other contest models is explored.

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ABSTRACT

I study the impact of asymmetric loss constraints on risk-taking behavior in the contest model of Seel and Strack (2013). I derive the unique Nash equilibrium outcome, the equilibrium payoffs and comparative statics about the bankruptcy risk.

1. Introduction

A contest is a simple and widely used mechanism in which each player's payoff is determined by his rank in a peer group. The main focus of the literature on contests has been on the trade-off between a higher effort cost and a higher chance of winning; see, e.g., Hillman and Samet (1987), Baye et al. (1996), or Siegel (2009, 2010). Applications of such contests include patent races, political campaigns, litigation, rent seeking, procurement and so forth.

In many contests in the financial industry, however, risk-taking is an important determinant of contest success. For example, think about competitions between private equity funds or mutual funds in which only the best performing funds receive substantial capital inflow in the next period or bonus payments for fund managers or CEO's if they outperform their peers. In order to focus on the risk-taking aspect, Seel and Strack (2013) proposed a contest model in which each player decides when to stop a privately observed Brownian motion and the player who stops his process at the highest value receives a prize. Thus, waiting longer entails the risk that the value of the process decreases.

During the last years, different versions of the model have been considered in the finance literature. For instance, Feng and Hobson (2015, in press-a,b) study the effects of regret-based agents, bidding caps, a random initial value, and different stochastic processes on the equilibrium outcome. In another interesting contribution, Fang and Noe (2015) establish an equivalence result between the stochastic contest model and a static model in which players choose a cumulative distribution function subject to a capacity constraint on the expected value. Moreover, they introduce multiple prizes and incomplete information and they analyze the probability of selecting more able contestants with this contest.

This paper contributes to the recent literature by focusing on another factor which influences risk-taking and which occurs in many applications in finance: contestants differ in their credit line, i.e., the maximal amount of money which they can lose. This introduces additional technical difficulties, since there are

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no simple boundary conditions as in Seel and Strack (2013); the equivalent model of Fang and Noe (2015) facilitates the analysis.

2. The model

I consider the following version of the model analyzed in Seel and Strack (2013). Each of two agents $i = 1, 2$ privately observes the realization of the stochastic process

$$X_t^i = x_0 + \sigma B_t^i,$$

where $x_0 > 0$ denotes the starting value and the random terms σB_t^i are independent Brownian motions scaled by $\sigma \in \mathbb{R}_+$.

Each player chooses a stopping time τ^i. The agents' stopping decision until time t has to be $\mathcal{F}_t^i$-measurable, where $\mathcal{F}_t^i = \sigma(\{X_s^i : s < t\})$ is the sigma algebra induced by the possible observations of the process X_s^i before time t. Additionally, I restrict stopping times in two ways: First, they should have finite expectation, i.e., $\mathbb{E}(\tau^i) < \infty$. The second restriction is the loss constraint. Without loss of generality, assume that player 2 has a tighter constraint and that he has to stop once the process hits zero, i.e., $\tau^i \le \inf\{t \in \mathbb{R}_+ : X_t^i = 0\}$ a.s.. Player 1 can make higher losses and thus he faces the weaker constraint $\tau^i \le \inf\{t \in \mathbb{R}_+ : X_t^i = \underline{x}\}$ a.s., where $\underline{x} < 0$ is the difference in the loss constraints.

Note that a stopping strategy induces a distribution over the values of the process at the stopping time, which I denote by $F_i(x) = \mathbb{P}(X_{\tau^i}^i \le x)$. The player who stops his process at the highest value wins a prize, which I normalize to one. Ties are broken randomly. Formally, the payoff/winning probability is

$$\pi_i = \mathbf{1}_{\{X_{\tau^i}^i > X_{\tau^j}^j\}} + \frac{1}{2}\mathbf{1}_{\{X_{\tau^i}^i = X_{\tau^j}^j\}}.$$

Each player maximizes the above winning probability.

Fang and Noe (2015) show that the stochastic contest model has the same Nash equilibrium distributions F_i as a model in which players choose their cumulative distribution functions subject to the constraint that the expected value of the underlying random variable is x_0^i. Including the loss constraints, player 2 thus faces the choice of a cumulative distribution F_2 on $[0, \infty)$ subject to the capacity constraint (expected value equals x_0). Player 1 chooses a cumulative distribution F_1 on $[\underline{x}, \infty)$ subject to the capacity constraint, where $-\underline{x} > 0$ is the amount of additional money which player 2 can lose.

3. Equilibrium characterization

In this section, I determine the Nash equilibrium of the contest. In equilibrium, player 1 must choose a cumulative distribution function F_1 which solves

$$\max_{dF_1 \ge 0} \int F_2(x) dF_1(x) \quad \text{s.t.} \int_{\underline{x}}^{\infty} x dF_1(x) = x_0$$

and F_2 must solve

$$\max_{dF_2 \ge 0} \int F_1(x) dF_2(x) \quad \text{s.t.} \int_0^{\infty} x dF_2(x) = x_0.$$

The constraint captures the available capacity (the expected value should be x_0). The following proposition characterizes the Nash equilibrium of the game:

Proposition 1. *In any Nash equilibrium, the cumulative distribution functions are*

$$F_1(x) = \begin{cases} 0 & \text{if } x < \underline{x}, \\ \alpha & \text{if } x \in [\underline{x}, 0), \\ \alpha + (1-\alpha)\frac{x}{\bar{x}} & \text{if } x \in [0, \bar{x}], \\ 1 & \text{if } x > \bar{x}, \end{cases}$$

and

$$F_2(x) = \begin{cases} 0 & \text{if } x < 0, \\ \beta + (1-\beta)\frac{x}{\bar{x}} & \text{if } x \in [0, \bar{x}], \\ 1 & \text{if } x > \bar{x}, \end{cases}$$

where

$$\alpha = \frac{x_0 - \sqrt{x_0^2 - 2x_0\underline{x}}}{2\underline{x} - x_0 - \sqrt{x_0^2 - 2x_0\underline{x}}},$$

$$\beta = \frac{-\underline{x}}{x_0 + \sqrt{x_0^2 - 2x_0\underline{x}} - \underline{x}},$$

and

$$\bar{x} = x_0 + \sqrt{x_0^2 - 2x_0\underline{x}}.$$

Proof. The proof is split into four steps: verifying that both functions are cumulative distributions, the capacity constraints, the best-response properties and uniqueness.

Step 1 (Cumulative Distribution Function): To be a cumulative distribution, F_1 and F_2 have to be non-decreasing, right-continuous functions with $\lim_{x \to -\infty} F_i = 0$ and $\lim_{x \to \infty} F_i = 1$ for $i = 1, 2$. Clearly, these conditions are satisfied if $\alpha \in (0, 1)$ and $\beta \in (0, 1)$. To see that $\alpha \in (0, 1)$, note that the numerator is negative since $x_0 - \sqrt{x_0^2 - 2x_0\underline{x}} < x_0 - \sqrt{x_0^2} = 0$. The denominator is also negative and has a larger absolute value than the numerator since $2\underline{x} - x_0 < x_0$. Thus, we obtain $\alpha \in (0, 1)$. Since both the numerator and denominator in the expression of β are positive with the denominator being larger than the numerator, we have $\beta \in (0, 1)$.

Step 2 (Capacity Constraint): To verify the capacity constraint, I calculate the expected value and equate it to the constraint for both F_2 and F_1 in order to see for which values the equality holds.

$$\int_0^{\bar{x}} x dF_2(x) = \frac{(1-\beta)\bar{x}}{2} = \frac{\left(1 - \left(\frac{-\underline{x}}{\bar{x} - \underline{x}}\right)\right)\bar{x}}{2} = \frac{\bar{x}^2}{2(\bar{x} - \underline{x})} = x_0.$$

Since $\bar{x} > x_0$, this yields $\bar{x} = x_0 + \sqrt{x_0^2 - 2x_0\underline{x}}$. Thus, $\beta = \frac{-\underline{x}}{\bar{x} - \underline{x}} = \frac{-\underline{x}}{x_0 + \sqrt{x_0^2 - 2x_0\underline{x}} - \underline{x}}$, i.e., the equality holds for the parameters given in the proposition.

For the first distribution, calculating the expected value setting it equal to x_0 yields

$$\int_{\underline{x}}^{\bar{x}} x dF_1(x) = \alpha\underline{x} + \frac{(1-\alpha)\bar{x}}{2}$$
$$= \alpha\underline{x} + \frac{(1-\alpha)(x_0 + \sqrt{x_0^2 - 2x_0\underline{x}})}{2} = x_0.$$

Thus, I obtain

$$\alpha = \frac{x_0 - \sqrt{x_0^2 - 2x_0\underline{x}}}{2\underline{x} - x_0 - \sqrt{x_0^2 - 2x_0\underline{x}}}.$$

Hence, for the parameter values in the proposition, both capacity constraints are satisfied.

Step 3 (Existence): In the next step, I verify that two distributions are mutual best responses.

First of all, let me argue that the support of any best response to F_1 has to be a subset of $[0, \bar{x}]$: towards a contradiction, for any