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Robust competitive auctions

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HIGHLIGHTS

- The Revelation Principle is not readily applicable in markets with many principals.
- Does a seller want to deviate to any mechanism in competing auctions (Peters 1997)?
- The sufficient condition for the robustness is embedded in its notion of equilibrium.

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ABSTRACT

A competitive distribution of auctions (Peters 1997) is robust to the possibility of a seller's deviation to any arbitrary mechanisms, let alone direct mechanisms because the sufficient condition for the robustness is embedded in its notion of equilibrium.

1. Introduction

Peters (1997) studies a decentralized auction market with many sellers and buyers, all of whom differ in terms of their valuation for the item being traded. To highlight frictions in the market, he focuses on an *incentive consistent continuation equilibrium* in which (i) buyers use the same strategy (symmetry) and (ii) they choose non-deviating sellers with equal probability if their auctions are the same (non-discrimination among non-deviating sellers). His result shows that when every seller offers a second price auction with reserve price equal to his cost, a seller cannot improve his profits by offering any alternative direct mechanism instead of his second price auction as the number of sellers increases to the infinity. However, it is not yet known whether a seller can gain by deviating to an arbitrary mechanism other than a direct mechanism.[1]

Fix the distribution of second price auctions offered by all sellers except a deviating seller. In any incentive consistent continuation equilibrium upon a seller's deviation to any arbitrary (indirect) mechanism, one can always extract a (Bayesian) incentive compatible direct mechanism from the deviating seller's mechanism by using the buyers' strategies of communicating with the deviating seller.[2] If the deviating seller directly deviates to that incentive compatible direct mechanism, one can derive a payoff-equivalent incentive consistent continuation equilibrium where the buyers maintain the original probabilities of selecting the deviating seller.

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[1] The standard Revelation Principle is not readily applicable to a market with many sellers (principals) because buyers (agents) are informed not only about their valuation but also about selling mechanisms offered by competing sellers. Therefore, the message space only over buyers' possible valuations in a direct mechanism may not be large enough to encompass all the information that buyers have in the market. Epstein and Peters (1999) propose a mechanism with the universal language that allows buyers to describe competing seller's selling mechanism and establish a revelation principle with this type of universal mechanisms. However, it is not easy to apply universal mechanisms.

[2] Regardless of the selling mechanism offered by the deviating seller, we can conveniently fix buyers' truthful type reporting to the other sellers upon selecting them because their selling mechanisms, second price auctions, are dominant strategy incentive compatible.

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Therefore, the deviating seller cannot gain by deviating to any arbitrary mechanism if he cannot gain in every incentive consistent continuation equilibrium upon offering every possible incentive compatible direct mechanism.

This sufficient condition is embedded in the notion of a competitive distribution of auctions in Peters (1997), so a competitive distribution of auctions is robust to the possibility that sellers may deviate to any arbitrary mechanisms, not just direct mechanisms. The key for the robustness is that for any given incentive compatible direct mechanism that a seller may deviate to, there may be multiple incentive consistent continuation equilibria, which differ in terms of the buyers' strategies of selecting him and hence we need to consider all possible selection strategies that make the deviator's direct mechanism incentive compatible in order to check if a seller has an incentive to deviate to any arbitrary mechanism.

2. A competitive distribution of auctions

In Peters (1997), J sellers face κJ buyers: $\mathcal{J} = \{1, \ldots, J\}$ and $\mathcal{I} = \{J+1, \ldots, (\kappa+1)J\}$. Each seller has one unit of an indivisible good to sell. Each buyer needs one unit of the good and the distribution of the buyer's valuation is denoted by F with its support $[0, 1]$. Each seller has a cost associated with selling his good. The distribution of costs in the population of sellers is denoted by G with its support contained in $[0, 1]$. In seller j's anonymous direct mechanism, the message space for each buyer is $\bar{X} = [0, 1] \cup \{x^\circ\}$, where x° denotes that the buyer does not participate in the mechanism. Suppose that x is a buyer's message and that $\mathbf{x}$ is the profile of the other buyers' messages. Let $p_j(x, \mathbf{x})$ denote the price that a buyer pays to seller j and $q_j(x, \mathbf{x})$ the probability with which a buyer acquires the object. Then, seller j's direct mechanism is denoted by $\mu_j = \{p_j, q_j\}$.

Suppose that π denotes a buyer's incentive consistent selection strategy.[3] In particular, $\pi(x, \mu_j, \mu_{-j}) \in [0, 1]$ specifies the probability with which a buyer with valuation x selects seller j who offers μ_j given the other sellers' mechanisms μ_{-j}. Given $\mu = (\mu_j, \mu_{-j})$ and π, let $z_j(\mu, \pi)(x)$ denote the probability that a buyer's valuation is less than x or selects a seller other than j. Then, $z_j(\mu, \pi)$ can be derived as follows[4]: for all $x \in [0, 1]$,

$$z_j(\mu, \pi)(x) = 1 - \int_x^1 \pi(s, \mu_j, \mu_{-j}) f(s) ds. \tag{1}$$

One can use $z_j(\mu, \pi)$ to derive the reduced-form probability $Q_j(x, \mu, \pi)$ with which a buyer with valuation x expects to acquire the object as follows:

$$Q_j(x, \mu_j, \pi) = \int_0^1 \cdots \int_0^1 q_j(x, s_{J+2}, \ldots, s_{(\kappa+1)J}) \times dz_j(\mu, \pi)(s_{J+2}) \ldots dz_j(\mu, \pi)(s_{(\kappa+1)J}). \tag{2}$$

Similarly we can derive $P_j(x, \mu_j, \pi)$, the reduced-form price that she expects to pay upon selecting seller j. Therefore, $z_j(\mu, \pi)$ determines the reduced-form mechanism, $Q_j(x, \mu_j, \pi)$ and $P_j(x, \mu_j, \pi)$.

Peters (1997) considers the finite approximation of the limit game with the infinite number of traders given the fixed ratio κ of buyers to sellers.[5] A cutoff valuation for seller j is the infimum of the set of valuations for which buyers choose seller j with positive probability. Let H denote a distribution of cutoff valuations for the limit game.

For a finite approximation, consider the market where $J - 1$ sellers hold *second-price auctions* $\bar{\mu}_{-J} = \{\bar{\mu}_1, \ldots, \bar{\mu}_{J-1}\}$ with the distribution of the cutoff valuations $\bar{H}_J$ that converges almost everywhere to H. Let $\pi_J'(x, \mu_J')$ be the selection probability with which a buyer with valuation x selects deviating seller J when his direct mechanism is μ_J' given $\bar{\mu}_{-J}$. According to Peters (1997), $\pi_J'(\cdot, \mu_J')$ alone completely determines the incentive consistent strategies of selecting non-deviating sellers.[6]

Then, one can calculate the payoff, $\bar{v}_1(x, \mu_J', J)$, to a buyer with valuation x by selecting the non-deviating seller offering the lowest reserve price:

$$\bar{v}_1(x, \mu_J', J) = \int_{y_1}^x \left[1 - \int_v^1 \frac{1 - \pi_J'(x, \mu_J')}{n_J(s, H)} f(s) ds\right]^{\kappa J - 1} dv, \tag{3}$$

where $n_J(x, H) = \max\{j : \bar{y}_j \leq x\}$ and $\frac{1-\pi_J'(x, \mu_J')}{n_J(s,H)}$ is the selection probability with which a buyer with valuation x chooses the non-deviating seller with the lowest reserve price.[7] Peters (1997) shows that every non-deviating seller will give the same expected payoff as the non-deviating seller who offers the lowest reserve price when the matching process is incentive consistent.

Then, for any incentive consistent selection strategy π_J', the deviating seller's payoff associated with offering an incentive compatible direct mechanism is

$$\hat{\Phi}_J(w, \mu_J', H, \pi_J') = w + \kappa J \int_0^1 \big[(x - w) Q_J(x, \mu_J', \pi_J') - \bar{v}_1(x, \mu_J', J)\big] \pi_J'(x, \mu_J') f(x) dx. \tag{4}$$

Finally, let $\hat{\Phi}_J'(w, \mu'(y), H)$ denote the payoff to the deviating seller with cost w if he offers a second-price auction $\mu'(y)$ that induces a cutoff valuation y. Let $\Pi_J'(\mu_J', H)$ be the set of all incentive consistent selection strategies π_J' that lead to an incentive consistent continuation equilibrium given (μ_J', H). Let $\mathcal{M}_J^B(H)$ be the set of all (Bayesian) incentive compatible direct mechanisms available for each seller's deviation given H. The definition of a competitive distribution of second-price auctions in Peters (1997) can be provided as follows:

Definition 1. A competitive distribution of second-price auctions is a distribution of cutoff valuations H and a cutoff rule $y : [0, 1] \to [0, 1]$ such that for almost all w,

1. for all $\mu_J' \in \mathcal{M}_J^B(H)$ and all $\pi_J' \in \Pi_J'(\mu_J', H)$
$$\lim_{J\to\infty} \hat{\Phi}_J'(w, \mu'(y(w)), H) \geq \lim_{J\to\infty} \hat{\Phi}_J(w, \mu_J', H, \pi_J') \tag{5}$$
2. and $H(y(w)) = G(w)$.

Theorem 5 in Peters (1997) shows that there is a competitive distribution of second-price auctions in which each seller offers a second-price auction with reserve price equal to his cost.

[3] An incentive consistent strategy implies a strategy that buyers use in an incentive consistent continuation equilibrium.

[4] If not confused, some of the notations are slightly modified from those in Peters (1997).

[5] This is because, as Peters and Severinov (1997) suggest, competing auction games often do not admit a pure-strategy equilibrium with the finite number of sellers. Burguet and Sákovics (1999) show the existence of mixed-strategy equilibrium in the two-seller case. Virág (2010) extends their result to any finite number of homogeneous sellers and shows that when sellers' costs are equal to zero, equilibrium reserve prices converge to zero.

[6] Unless specified, we will use π_J' when we refer to the buyer's incentive consistent selection strategy.

[7] See Lemma 2 in Peters (1997) for details.