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Group inefficiency in a common property resource game with asymmetric players

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HIGHLIGHTS

- Cooperative solutions are studied in a dynamic game with asymmetric discounting.
- Time-consistent cooperative solutions can be group inefficient.
- Joint payments for all players can be higher if cooperation is forbidden.
- Noncooperative equilibria can be time inconsistent if cooperation is allowed.

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ABSTRACT

If the joint preferences of asymmetric players having different discount rates are represented by the sum of intertemporal utilities, they become time-inconsistent. It is shown how time-consistent solutions for this problem can be strongly inefficient: the sum of payoffs can be higher if cooperation or coordination is forbidden than if it is allowed.

1. Introduction

In a dynamic game, there is no reason to believe that consumers, firms or countries have identical time preferences for utility streams. In that case, if there is a social planner who aggregates the preferences of all agents by simply adding their intertemporal utility functions, or if the economic agents decide to communicate and coordinate their strategies in order to optimize their collective payoff, what should they maximize? Many papers have addressed the issue of aggregating preferences (see Ekeland and Lazrak, 2010; Ekeland et al., 2015; Karp, 2015, and references therein). Unfortunately, in this framework, a problem of time-consistency arises. What is optimal for the coalition or the society at time t will be no longer optimal at time s, for $s > t$. In the derivation of time-consistent policies (de-Paz et al., 2013; Ekeland et al., 2013), as in nonconstant discounting, solutions are computed by finding subgame perfect equilibria in a noncooperative sequential game where the players are the different t-coalitions. However, as we show in this paper, these equilibria can be group inefficient: payments to all the t-coalitions can be lower than the sum of payments if agents play a noncooperative game. Our result is a consequence of the time-inconsistency of the joint preferences (in a different context with non-constant discounting, Krusell et al., 2002; Hiraguchi, 2014 found a related result).

We briefly review the time-consistent solution studied in de-Paz et al. (2013). Let N be the number of players, and $x \in X \subset \mathbf{R}^n$ the state variables. For each player $i \in \{1, \ldots, N\}$, let $c_i \in U_i \subset \mathbf{R}^{m_i}$ be her control (decision) variables, $u_i(x, c)$ her instantaneous utility function ($c = (c_1, \ldots, c_N)$), and $\rho_i \geq 0$ her discount rate. The time horizon τ can be finite or infinite. In the latter case,

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integrals are assumed to converge. The intertemporal utility function for Player i at time t is

$$J_i(x, c_1, \ldots, c_N, t) = \int_t^\tau e^{-\rho_i(s-t)} u_i(x(s), c_1(s), \ldots, c_N(s))\, ds, \quad \text{with}$$
$$\dot{x}(s) = g(x(s), c_1(s), \ldots, c_N(s)),\ x(t) = x_t. \tag{1}$$

Functions u_i and g are C^1. In a cooperative setting, we aggregate preferences as

$$J^c = \sum_{i=1}^N J_i = \sum_{i=1}^N \int_t^\tau e^{-\rho_i(s-t)} u_i\left(x(s), c_1(s), \ldots, c_N(s)\right) ds. \tag{2}$$

Since the discount rates are different, the joint time preferences are time inconsistent. In order to find time-consistent solutions, the problem is solved as a noncooperative sequential game with an continuum number of players (each player is the coalition at time t, for $t \in [0, \tau]$).

Following Ekeland and Lazrak (2010) (see also Karp, 2007), if $c^*(s) = \phi(x(s), s)$ is the equilibrium rule for (2) subject to (1), by denoting $x_t = x$, the value function is

$$V(x, t) = \sum_{i=1}^N \int_t^\tau e^{-\rho_i(s-t)} u_i(x(s), \phi(x(s), s))\, ds.$$

Next, for $\epsilon > 0$ and $\bar{c} = (\bar{c}_1, \ldots, \bar{c}_N)$, $\bar{c}_i \in U_i$, let

$$c_\epsilon(s) = \begin{cases} \bar{c} & \text{if } s \in [t, t+\epsilon], \\ \phi(x(s), s) & \text{if } s > t+\epsilon. \end{cases} \tag{3}$$

If the t-coalition has the ability to precommit its behavior during the period $[t, t+\epsilon]$, the valuation along the perturbed control path c_ϵ is given by

$$V_{c_\epsilon}(x, t) = \sum_{i=1}^N \left\{ \int_t^{t+\epsilon} e^{-\rho_i(s-t)} u_i(x(s), \bar{c}) ds + \int_{t+\epsilon}^\tau e^{-\rho_i(s-t)} u_i(x(s), \phi(x(s), s)) ds \right\}.$$

Definition 1. A decision rule $c^*(s) = \phi(x(s), s)$ is called a t-cooperative equilibrium (t-CE) if, for any admissible c_ϵ given by (3),

$$\lim_{\epsilon \to 0^+} \frac{V(x, t) - V_{c_\epsilon}(x, t)}{\epsilon} = P(x, \phi, \bar{c}, t) \ge 0.$$ [1]

In de-Paz et al. (2013) it is proved that, if a decision rule $c = \phi(x, t)$ is such that the functions

$$V_i(x, t) = \int_t^\tau e^{-\rho_i(s-t)} u_i(x(s), \phi(x(s), s))\, ds \tag{4}$$

are of class C^1 and

$$\phi(x, t) = \arg\max_{\{c\}} \left\{ \sum_{i=1}^N u_i(x, c) + \sum_{i=1}^N \nabla_x V_i(x, t) \cdot g(x, c) \right\}, \tag{5}$$

then $c = \phi(x, t)$ is a t-CE. If $\tau = \infty$, the value functions V_i become time-independent.

[1] Note that the maximum in $\bar{c}$ of $P(x, \phi, \bar{c}, t)$ is achieved for the equilibrium rule $\bar{c} = \phi(x, t)$.

2. Group inefficiency

The t-cooperative equilibrium is the natural solution that we obtain when we search for time-consistent solutions to the problem of aggregating preferences of decision makers by simply taking the sum of their intertemporal utilities. However, as we show in this section with an example, there are cases in which this solution is group inefficient.

We consider the following basic problem of exploitation of a nonrenewable natural resource. If $x(t)$ represents the resource's stock and $c_i(t)$ the extraction rate of Player i ($i = 1, \ldots, N$), the state dynamics is

$$\dot{x}(s) = -\sum_{i=1}^N c_i(s), \quad x(t) = x_t, \tag{6}$$

and the intertemporal utility function of Player i is

$$J_i = \int_t^\infty e^{-\rho_i(s-t)} \ln\left[(c_i(s))^{\mu_i}\right] ds, \quad \mu_i > 0.$$

Note that we have introduced asymmetries both in the discount functions ($\rho_i \neq \rho_j$, in general) and in the utility functions $u_i(c_i) = \ln c_i^{\mu_i}$ (if $\mu_i \neq \mu_j$).

We restrict our attention to the existence of linear equilibria. First, in a noncooperative setting, linear (feedback) MPNE are given by $c_i^n(x) = \rho_i x$, with value functions

$$W_i(x) = \frac{\mu_i}{\rho_i} \ln x + \frac{\mu_i}{\rho_i} \ln \rho_i - \frac{\mu_i}{\rho_i^2} \sum_{j=1}^N \rho_j. \tag{7}$$

Note that, if Player i's opponents use the strategy $\phi_j^n(x) = \alpha_j x$, MPNE are found by solving the HJB equation $\rho_i W_i(x) = \max_{\{c_i\}}[\mu_i \ln c_i - W_i'(x)(c_i + \sum_{j \neq i} \alpha_j x)]$. From the maximization problem, $c_i = \frac{\mu_i}{W_i'(x)}$. The HJB equation becomes $\rho_i W_i(x) = \mu_i \left(\ln \mu_i - \ln W_i'(x) - 1\right) - (\sum_{j \neq i} \alpha_j) x W_i'(x)$. It can be easily checked that, for $\alpha_j = \rho_j$, function $W_i(x)$ given by (7) verifies this equation.

For the calculation of t-CE, we have to solve (4)–(5). We look for the solution to $\max_{\{c_1, \ldots, c_N\}} \left\{ \sum_{j=1}^N \mu_j \ln c_j - \left(\sum_{i=1}^N V_i'(x)\right) \left(\sum_{j=1}^N c_j\right) \right\}$. Therefore, $c_j^c = \mu_j \left(\sum_{i=1}^N V_i'(x)\right)^{-1}$. In order to derive linear equilibria, we search for value functions of the form $V_i(x) = \alpha_i^c \ln x + \beta_i^c$, for $i = 1, \ldots, N$. Then $c_j^c = \phi_j^c(x) = \mu_j x / (\sum_{i=1}^N \alpha_i^c)$. By substituting in (6) and solving we obtain $x(s) = x_t \exp\left[-\frac{\sum_{i=1}^N \mu_i}{\sum_{j=1}^N \mu_j/\rho_j}(s-t)\right]$. Hence, $\ln \phi_i^c(x(s)) = \ln x_t - \frac{\sum_{j=1}^N \mu_j}{\sum_{j=1}^N \mu_j/\rho_j}(s-t)$. Since, from (4), $V_i(x) = \int_t^\infty e^{-\rho_i(s-t)} \mu_i \ln \phi_i^c (x(s))\, ds$ with $x_t = x$, then

$$\alpha_i^c \ln x + \beta_i^c = \left[\int_t^\infty e^{-\rho_i(s-t)}\, ds\right] \mu_i \ln x - \mu_i \frac{\sum_{j=1}^N \mu_j}{\sum_{j=1}^N \mu_j/\rho_j} \int_t^\infty e^{-\rho_i(s-t)} (s-t)\, ds.$$

By solving and simplifying we finally obtain

$$c_i^c(x) = \frac{\mu_i}{\sum_{j=1}^N \mu_j/\rho_j} x \quad \text{and}$$