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Fertility and unemployment in a social security system

Leran Wang [*]

Graduate School of Economics, Keio University, 2-15-45 Mita, Minato-ku, Tokyo, 108-8345, Japan

HIGHLIGHTS

- A two-period OLG model with endogenous fertility and unemployment are considered.
- A social security system is consisted of a PAYG pension and child allowances.
- Minimum wage is set constant by law in this model.
- High-level pensions increase fertility and decrease unemployment.
- Low-level pensions worsen fertility and unemployment even with child allowances.

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ABSTRACT

This study analyzes how endogenous fertility and unemployment are affected by a social security system in an overlapping generations model. The analysis reveals that for any given minimum wage, the pension may improve fertility and decrease unemployment.

1. Introduction

The problems of fertility reduction and unemployment increase have puzzled many governments over the past decades. Can the widely used social security system help solve these problems?

Analyses concerning fertility consider children as a consumption good or an investment good. The former, the altruism hypothesis, argues that parents procreate because they derive satisfaction from raising children (Barro and Becker, 1989), while the latter, the egoism hypothesis, insists that parents raise children because of old-age security considerations (Cigno, 1993). Concerning the effects of pension on fertility, many empirical studies believe that egoism prevails over altruism (e.g., Cigno and Rosati, 1992 and Hohm, 1975). However, few studies consider that people have incentives to raise more children when their future lives are guaranteed by pensions.

Fanti and Gori (2007) find that introducing child allowances reduces the fertility rate by obstructing capital accumulation and increasing the unemployment rate for any given minimum wage value. The present study extends the overlapping generations (OLG) model by incorporating not only child allowances but also a pay-as-you-go (PAYG) public pension into the economy and examines the effects of the two subsidies on endogenous fertility and unemployment using comparative statics.

The remainder of the paper is organized as follows: Section 2 introduces the model; Section 3 discusses the equilibria of the two endogenous variables; Section 4 reviews the comparative statics; and Section 5 concludes.

[*] Tel.: +81 80 4843 0925.
E-mail address: orakuzenn@gmail.com.

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2. The model

2.1. The government

Taxes from workers' income are used to finance the government's public pension and child allowances[1]:

$$\pi_t \omega L_t = \theta N_{t-1} + \varphi n_t N_t, \tag{1}$$

where π_t is the income tax rate of social security involving the pension and the child allowances; ω is the constant minimum wage, which is set to exceed the competitive wage[2]; N_t is the population of generation t; L_t is the labor force, and $L_t = N_t(1-u_t)$, where u_t is the unemployment rate. The relationship between populations of adjacent generations is linked by the endogenous fertility as $N_{t+1} = N_t n_t$.

2.2. Consumption

Consider a two-period general equilibrium OLG model in a closed economy. Individuals gain utility from youth consumption, old-age consumption, and child rearing:

$$U\left(c_{1,t}, c_{2,t+1}, n_t\right) = \alpha \ln\left(c_{1,t}\right) + \beta \ln\left(c_{2,t+1}\right) + \gamma \ln\left(n_t\right), \tag{2}$$

where $\alpha, \beta, \gamma \in (0,1)$ are the utility weights, and $\alpha + \beta + \gamma = 1$. n_t is the number of children. The budget constraint of the young in generation t is

$$c_{1,t} + s_t = \omega(1-u_t)(1-\pi_t) - (m-\varphi) n_t \tag{3}$$

where m and φ are the average cost of raising children and the child allowances level ($0 < \varphi < m < \omega$). In the youth period, the wages of workers support consumption ($c_{1,t}$), savings (s_t), social security payments and the cost of raising children. In the retirement period, old-age consumption $\left(c_{2,t+1}\right)$ comes from savings ($s_t R_{t+1}$) and the PAYG pension (θ, $0 < \theta < \omega$):

$$c_{2,t+1} = s_t R_{t+1} + \theta, \tag{4}$$

where $R_{t+1} = 1 + r_{t+1}$, and r_{t+1} is the interest rate in period $t+1$. According to the utility maximization,

$$s_t = \beta\omega(1-u_t) - \beta\varphi n_t - \frac{\beta\theta}{n_{t-1}} - \frac{(1-\beta)\theta}{R_{t+1}}, \tag{5}$$

$$n_t = \frac{\gamma}{m - \varphi + \gamma\varphi}\left[\omega(1-u_t) - \frac{\theta}{n_{t-1}} + \frac{\theta}{R_{t+1}}\right]. \tag{6}$$

2.3. Production

It is assumed that innumerable identical firms act competitively. The Cobb–Douglas production function is $Y_t = K_t^{\delta} L_t^{1-\delta}$, where K_t and L_t denote the capital and labor input, and $\delta \in (0,1)$ represents the weight of the capital input. To maximize profits,

$$\omega = (1-\delta)\left(\frac{K_t}{L_t}\right)^{\delta}, \tag{7}$$

$$r_t = \delta\left(\frac{K_t}{L_t}\right)^{\delta-1} - 1. \tag{8}$$

Therefore, the capital–labor ratio and the interest factor are constant.[3] As $L_t = N_t(1-u_t)$,

$$\frac{K_t}{L_t} = \frac{k_t}{1-u_t},$$

where $k_t = K_t/N_t$. Thus,

$$u_t = 1 - k_t\left(\frac{1-\delta}{\omega}\right)^{\frac{1}{\delta}}. \tag{9}$$

2.4. The capital market

The equilibrium of the capital market gives $S_t = K_{t+1}$. Therefore,

$$\frac{s_t}{n_t} = k_{t+1}. \tag{10}$$

Substituting Eqs. (5) and (6) into Eq. (10), the per-capita capital is

$$k_{t+1} = \frac{\beta(m-\varphi)}{\gamma} - \frac{\theta\varepsilon}{\delta n_t}. \tag{11}$$

This implies that capital accumulation is obstructed because of pension enforcement.[4] When the fertility rate increases, the population bearing the pension tax burden increases. When the tax burden per capita decreases, the savings level increases, and the next-period capital accumulation per capita is fostered, indicating that the population alteration positively affects the per-capita capital accumulation.

3. Equilibrium

Substituting Eq. (11) into (9), and then into Eq. (6), the dynamic fertility equilibrium is

$$n_t = \frac{\gamma}{m - \varphi + \gamma\varphi}\left[\frac{\beta(m-\varphi)(1-\delta)}{\gamma\varepsilon} + \frac{\theta\varepsilon}{\delta} - \frac{\theta}{\delta n_{t-1}}\right], \tag{12}$$

where $\varepsilon := \omega^{\frac{1-\delta}{\delta}}(1-\delta)^{\frac{\delta-1}{\delta}}$. Substituting Eq. (11) into (9), and replacing n_t and n_{t-1} with u_{t+1} and u_t, and then into Eq. (12), the unemployment equilibrium dynamics are

$$u_{t+1} = \frac{\theta\sigma^2\varepsilon^2(m - \varphi + \gamma\varphi)}{\sigma\left[\theta\gamma\varepsilon^2 - \beta(m-\varphi)\delta^2\right] + \gamma\delta(1-u_t)} + \frac{\gamma - \beta(m-\varphi)\sigma}{\gamma}, \tag{13}$$

where $\sigma = (1-\delta)^{\frac{1}{\delta}}\omega^{-\frac{1}{\delta}}$. Equilibria of the two endogenous variables are determined by Eqs. (12) and (13).[5]

When the fertility rate (n_{t-1}) increases, the per-capita capital accumulation (k_t) increases. As the capital–labor ratio is constant, the labor demand ($L_{t,d}$) increases. Then, the unemployment rate (u_t) decreases, which means that the labor supply ($L_{t,s}$) is increasing. Consequently, production (Y_t) and the fertility rate (n_t) increase. Thus, the per-capita capital (k_{t+1}) increases, and the unemployment rate (u_{t+1}) decreases.

4. Comparative statics

4.1. Public pension effect

Proposition 1. *For any given minimum wage value, when the pension level is higher than* $\frac{\delta}{\gamma\varepsilon^2}\left(2A - B + 2\sqrt{A(A-B)}\right)$, *it improves the fertility rate and decreases the unemployment rate; when the*

1 When the model was first set, the unemployment benefits were also considered. However, these had no effect on the final result, and they were thus removed from the model for simplicity.

2 See Fanti and Gori (2007, 2010).

3 $1 + r_t = \delta\left(\frac{K_t}{L_t}\right)^{\delta-1} = \frac{\delta}{\varepsilon}$, where $\varepsilon := \omega^{\frac{1}{\delta}-1}(1-\delta)^{1-\frac{1}{\delta}}$.

4 If there is no pension, as shown by Fanti and Gori (2007), the capital per capita remains unchanged.

5 There are three equilibria in each dynamics, but this study focuses on the neighborhoods of the stable and economically meaningful steady states.