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Behavior in the centipede game: A decision-theoretical perspective

Pavlo Blavatskyy *

School of Management and Governance, Murdoch University, 90 South Street, Murdoch, WA 6150, Australia

H I G H L I G H T S

- Behavior in the centipede game when players are not expected utility maximizers.
- Players choose under uncertainty in a probabilistic manner.
- A core deterministic decision theory is embedded in a model of probabilistic choice.
- We consider, inter alia, a constant error/trembles and quantal response equilibrium.
- Players adopt non-linear decision weights/overweight the likelihood of rare events.

A R T I C L E I N F O

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The centipede game is a two-player finite game of perfect information where a unique subgame perfect Nash equilibrium appears to be intuitively unappealing and descriptively inadequate. This paper analyzes behavior in the centipede game when a traditional game-theoretical assumption that players maximize expected utility is relaxed. We demonstrate the existence of a descriptively adequate subgame perfect equilibrium under two standard decision-theoretical assumptions. First, players choose under uncertainty in a probabilistic manner as captured by embedding a core deterministic decision theory in a model of probabilistic choice. Second, players adopt non-linear decision weights and overweight the likelihood of rare events as captured, for example, by rank-dependent utility or prospect theory.

1. The centipede game

The centipede game is a famous finite game of perfect information. In this game two players move sequentially one after another. At the beginning of the game, player 1 can either terminate the game immediately (in which case both players receive a small payoff of 1) or pass the move to player 2. Player 2 can then either terminate the game (in which case player 1 receives nothing and player 2 receives a payoff of 2) or pass the move back to player 1. Player 1 can then either terminate the game (in which case both players receive a payoff of 2) or pass the move back to player 2. The game continues in this fashion for many rounds with payoffs gradually increasing. If the last round is reached, player 2 must decide between option R where both players receive a payoff of 100 and option D where player 1 receives a payoff of 98 and player 2 receives a payoff of 101.

The centipede game is presented in the extensive form in Fig. 1. The idea of this game can be traced back to Rosenthal (1981, Fig. 3, p. 96). In all Nash equilibria of the centipede game player 1 chooses D in the first decision node. In a unique subgame perfect Nash equilibrium both players choose D in all decision nodes, which can be established by backward induction. This outcome appears counterintuitive—both players can get much higher utility in the later nodes if they do not terminate the game in the first node. Experimental evidence (*e.g.*, McKelvey and Palfrey, 1992) suggests that most people do not stop at the first node but terminate the game at some intermediate node (the game is rarely played till the last node).

2. Introducing models of probabilistic choice

Traditional game theory assumes that players maximize expected utility (*e.g.*, von Neumann and Morgenstern, 1944). In

* Tel.: +61 08 9360 2838; fax: +61 08 9360 6966.
E-mail address: P.Blavatskyy@murdoch.edu.au.

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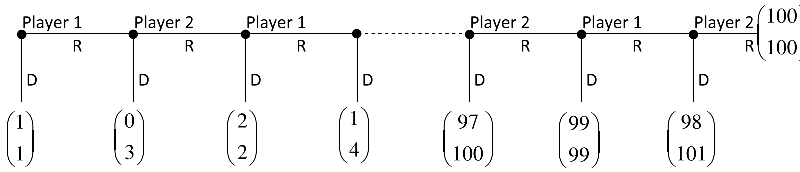

Fig. 1. The centipede game.

expected utility theory a decision maker chooses in a deterministic manner (except for a special case of indifference). Numerous laboratory experiments, however, establish that revealed choices under uncertainty are often probabilistic, *e.g.* Hey and Orme (1994) and Ballinger and Wilcox (1997). Thus, a more descriptively adequate modeling approach is to embed a deterministic decision theory (expected utility or a generalized non-expected utility theory) in a model of probabilistic choice (*cf.* Loomes and Sugden, 1998 and Blavatskyy and Pogrebna, 2010).

In traditional game theory players always pick the strategy that yields the highest expected utility. In this section we only assume that players are more likely to pick the strategy that yields the highest expected utility (but they do not necessarily always choose this strategy). With a model of probabilistic choice as a primitive we have a natural foundation for a mixed strategy equilibrium. In this equilibrium, players randomize not to keep the opponent indifferent but because it is in their nature to select better choice options with a higher probability but not all the time.

Several models of probabilistic choice were proposed in the literature but not all of them are promising candidates for analyzing the centipede game. Random utility or random preferences *e.g.* Falmagne (1985),[1] the models of Fishburn (1978) and Blavatskyy (2007, 2011) assume that choice under certainty is deterministic. The implications of these models are the same as in the subgame perfect Nash equilibrium described in Section 1. In the last node player 2 decides between option D that yields utility of 101 with certainty and option R that yields utility of 100 with certainty. If choice under certainty is deterministic, player 2 always chooses D in the last node. Knowing this, player 1 always chooses option D in the before–last node and so forth until we arrive at the conclusion that player 1 choses option D in the first node of the game.

One of the simplest models not assuming deterministic choice under certainty is a constant error or tremble model (*e.g.*, Harless and Camerer, 1994). In this model, a decision maker chooses the option with a higher expected utility with probability $1-\tau$ and the option with a lower expected utility—with probability $\tau \in (0, 0.5)$. Thus, in the last node of the centipede game, player 2 chooses option D with probability $1-\tau$ and option R—with probability τ. Knowing this, in the before–last node of the game, player 1 chooses option D with probability $1-\tau$ and option R—with probability τ. Going by backward induction we establish that in all nodes both players chose option D with probability $1-\tau$ and option R—with probability τ. Thus, a constant error/tremble model predicts that the centipede game most likely ends in the first node but the play might also terminate in one of the intermediate nodes or even in the last node (though the chances are small).

A more sophisticated model of probabilistic choice is Fechner (1860) model of random errors.[2] In this model, a decision maker chooses option D over option R with probability

$$P(D, R) = \Phi_{0,\sigma}(U(D) - U(R)) \tag{1}$$

where $\Phi_{0,\sigma} : \mathbb{R} \to [0, 1]$ is the cumulative distribution function of the normal distribution with zero mean and constant variance $\sigma > 0$ and $U : \{R, D\} \to \mathbb{R}$ is the expected utility of the corresponding choice option. Thus, in the last node of the centipede game, player 2 chooses option D with probability $\Phi_{0,\sigma}(1) > 0.5$ and option R with probability $\Phi_{0,\sigma}(-1) < 0.5$. Knowing this, in the before–last node, player 1 chooses option D with probability $\Phi_{0,\sigma}\left(2 \cdot \Phi_{0,\sigma}(1) - 1\right) > 0.5$ and option R with probability $\Phi_{0,\sigma}\left(1 - 2 \cdot \Phi_{0,\sigma}(1)\right) < 0.5$.[3] Proceeding by backward induction we establish that the probability of choosing R increases for both players (moving from the last to the first node) but never exceeds 0.5. Dashed lines in Figs. 2 and 3 plot the probability of choosing R in a subgame perfect equilibrium derived from the Fechner model with $\sigma = 1$ for players 1 and 2 respectively.

Luce (1959) choice model assumes that people first detect and delete dominated alternatives. Second, they chose in a probabilistic manner among the remaining non-dominated alternatives. The prediction of this model coincides with the subgame perfect Nash equilibrium described in Section 1.

Yet, in most microeconomic applications, the first stage of Luce's choice model is typically ignored and the mathematical formula of the second stage is applied to all choice alternatives. In application to game theory Luce's choice model is known as logit quantal response equilibrium (McKelvey and Palfrey, 1995). A decision maker chooses option D over option R with probability

$$P(D, R) = \frac{e^{\lambda U(D)}}{e^{\lambda U(D)} + e^{\lambda U(R)}} \tag{2}$$

where $\lambda > 0$ is a noise parameter. Models (1) and (2) generate nearly identical choice patterns. In a logit quantal response equilibrium the probability of choosing R increases (but never exceeds 0.5) for both players as we move from the last to the first node. Gray lines in Figs. 2 and 3 plot the probability of choosing R in a logit quantal response equilibrium with $\lambda = 1.6$ for players 1 and 2 respectively.

Wilcox (2008, 2011) recently proposed "contextual utility" model of probabilistic choice. When one choice option first-order stochastically dominates the other option, Wilcox (2011, p. 94) assumes that a decision maker chooses as if making a constant error/tremble: the dominant option is chosen with probability $1 - \omega/2$ and the dominated option is chosen with probability $\omega/2$, for small probability $\omega \in (0, 0.5)$. In the centipede game, this happens only when player 2 decides in the last decision node. In all other nodes of the centipede game neither option stochastically dominates the other option. In this case, Wilcox (2011, p. 96) assumes that a decision maker chooses option D over R with probability

$$P(D, R) = (1-\omega)\,\Phi_{0,\sigma}\left(\frac{U(D) - U(R)}{\overline{u} - \underline{u}}\right) + \frac{\omega}{2} \tag{3}$$

where $\overline{u}$ denotes the highest utility payoff that a player can receive in choice options D and R and $\underline{u}$ denotes the lowest utility payoff that a player can receive in choice options D and R.

[1] See also Loomes and Sugden (1995) for an application to decision theory and Gul and Pesendorfer (2006) for a behavioral characterization.

[2] See Hey and Orme (1994) for an application to decision theory and Blavatskyy (2008) for a behavioral characterization.

[3] We make a simplifying assumption that both players have the same variance $\sigma > 0$ of random errors.