Economics Letters 137 (2015) 32–35

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Centralized vs decentralized contests

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HIGHLIGHTS

- We compare decentralized and centralized contests.
- The contest is described by the Contest Success Function of Beviá–Corchón (2015).
- Total effort in both contests depends on a parameter that represents competition.
- Payoffs are not monotonic with the size of the contest.

ARTICLE INFO

Article history:
Received 29 July 2015
Received in revised form
3 October 2015
Accepted 16 October 2015
Available online 24 October 2015

JEL classification:
C72
D72
D74

Keywords:
Centralization
Decentralization
Contests

ABSTRACT

We compare two contests, decentralized in which there are several independent contests with non overlapping contestants and centralized in which all contestants fight for a unique prize which is the sum of all prizes in the small contests. We study the relationship between payoffs and efforts between these two contests.

1. Introduction

Is it better to gather all rent-seeking activities in one place, say Washington DC or Brussels, available to all citizens, rather than having them scattered all over US/EU and available only to the local people? Shall research funds for, say, economics be allocated in a single large contest available to all or shall they be allocated in several small contests only available to the local people?

These kind of questions arise again and again and they involve issues of efficiency and fairness. In this note we concentrate on an important aspect of the problem namely equilibrium payoffs and efforts spent by the contestants. Effort is sometimes socially valuable, such as when it is a proxy of the quality of the job to be done by the contest winner, or is sometimes a waste from the social welfare perspective, like rent seeking efforts aiming at a monopoly franchise. The effect on effort of passing from a large contest to a several small ones is not obvious. On the one hand the small contest has less competitors so individual efforts must increase. But on the other hand the prize is now smaller which calls for less effort.

In this note, we characterize the relationship of efforts in decentralized and in centralized contests assuming the Contest Success Function (CSF) proposed by Beviá and Corchón (2015) which generalizes Tullock CSF. We also find necessary and sufficient conditions for the contestants or the contest organizer to prefer centralized or decentralized contests.

The only paper dealing with this problem is by Wärneryd (2001). He assumes a generalized Tullock CSF and identical agents. Only our result on aggregate effort Proposition 2 is comparable to the results obtained by him, see footnote 4.

2. The model

In a contest, m agents called contestants spend efforts (bids) denoted by G_i in order to win a prize of value V_i. We consider two type of contests.

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http://dx.doi.org/10.1016/j.econlet.2015.10.022
0165-1765/

– **Decentralized** (**D**) k independent identical contests with n contestants each (thus $m = n$) and a prize valued as V_i, $i = 1, 2, \ldots, n$.
– **Centralized** (**C**) A single contest which is the aggregation of k identical contests. There are kn agents (thus $m = kn$) and a single prize valued as kV_i, $i = 1, 2, \ldots, kn$.[1]

A Contest Success Function (CSF) maps efforts of the agents into the probability that they will obtain the prize (or her share of the prize). Let $G = (G_1, \ldots, G_m)$. In a previous paper we introduced the idea of a notional CSF which maps G into real numbers (Beviá and Corchón, 2015). We proposed the following notional CSF:

$$f_i(G) = \alpha + \beta \frac{G_i - s\frac{\sum_{j\neq i} G_j}{m-1}}{\sum_{j=1}^{n} G_j}, \quad i \in \{1, \ldots, m\}$$

$$\text{if } \sum_{j=1}^{n} G_j \neq 0,\ \alpha \in [0,1],\ \beta \geq 0. \tag{2.1}$$

$$f_i(G) = \frac{1}{m} \quad \text{if } \sum_{j=1}^{m} G_j = 0,\ i \in \{1, \ldots, m\}. \tag{2.2}$$

This (notional) CSF mixes proportional CSF (Tullock, 1980) and (relative) difference CSF (Hirshleifer, 1989; Baik, 1998; Che and Gale, 2000). To convert this notional CSF into a CSF we first need that $\sum_{j=1}^{m} f_i(G) = 1$. This is accomplished by the following condition:

$$1 = m\alpha + \beta(1-s). \tag{2.3}$$

When $s = \alpha = 0$, $\beta = 1$ we have the Tullock CSF and $f(\cdot)$ is non negative. When $s \neq 0$ or $\alpha \neq 0$ non negativity is achieved when $m = 2$ by introducing max min operators as in Che and Gale (2000) or for general m by introducing a rationing rule which mimics the working of the CSF, see Beviá and Corchón (2015) for details. We show that in equilibrium there is no rationing so we leave the details of the rationing scheme to the interested reader. Let $h(\cdot)$ the CSF derived from (2.1) and (2.2) by taking into account that the range of such a function must yield probabilities. Consider a game in which strategies are expenses and payoff functions are

$$\pi_i = h_i(G)V_i - G_i. \tag{2.4}$$

Let $Y_i \equiv V_i \sum_{j=1}^{m} \frac{1}{V_j}$. To simplify the presentation, we focus on Nash equilibria in pure strategies in which all players exert a positive effort, which is guaranteed if:

$$Y_i > m - 1, \quad i \in \{1, \ldots, m\}, \tag{2.5}$$

which holds in the symmetric case where all valuations are identical and thus $Y_i = m$ for all i and when $m = 2$.[2] Suppose a D contest with values $(Y_1, Y_2, \ldots, Y_n)$. Then the corresponding values in the D contest are $(kY_1, kY_2, \ldots, kY_n)$. Thus, in D and C contests (2.5) reads

$$Y_i > n - 1, \quad i \in \{1, \ldots, n\}, \tag{2.6}$$

$$Y_i > \frac{kn-1}{k}. \tag{2.7}$$

Note that (2.7) implies (2.6) so we will only use the former.

To prove the existence of a Nash Equilibrium, we need the following assumption.

$$Y_i(\alpha+\beta) \geq \beta(m-1+s)\left(2 - \frac{m-1}{Y_i}\right),$$
$$i \in \{1, \ldots, m\}. \tag{2.8}$$

When all players have identical valuations, (2.8) is $m \geq \beta(m-1+s)$. In Beviá and Corchón (2015) we prove that a sufficient condition for (2.8) is $\alpha + \beta \leq 1$. In the Tullock case (2.8) also holds. Taking into account (2.3), (2.8) is

$$Y_i \geq \beta(m-1+s)\left(m\left(2 - \frac{m-1}{Y_i}\right) - Y_i\right),$$
$$i \in \{1, \ldots, m\}. \tag{2.9}$$

In Beviá and Corchón (2015) we proved the following:

Lemma 1. *Under (2.5) and (2.8) there is a Nash Equilibrium $(G_i^*)_{i=1}^n$ such that:*

$$G_i^* = \frac{\beta(m-1+s)V_i}{Y_i}\left(1 - \frac{m-1}{Y_i}\right), \quad i \in \{1, \ldots, m\}, \tag{2.10}$$

$$\pi_i^* = \frac{V_i}{Y_i}\left((\alpha+\beta)Y_i - \beta(m-1+s)\left(2 - \frac{m-1}{Y_i}\right)\right),$$
$$m = n, kn. \tag{2.11}$$

We proved Lemma 1 in Beviá and Corchón (2015) by constructing an auxiliary game in which payoff functions are $f_i(G)V_i - G_i$. This game has a unique Nash equilibrium characterized by first order conditions (FOC) of payoff maximization:

$$\beta V_i \frac{\sum_{j\neq i} G_j \left(1 + \frac{s}{m-1}\right)}{\left(\sum_{j=1}^{m} G_j\right)^2} - 1 = 0, \quad i \in \{1, \ldots, m\}. \tag{2.12}$$

We showed that FOC hold with equality for all agents and so (2.12) yields (2.10). Thus if payoffs are non negative at (2.10) this is indeed an equilibrium. And the condition for this is (2.9).

We now study how equilibrium effort changes when we pass from a small contest with n agents to a large contest with kn contestants and β and s do not change.[3] This is because β and s are the two parameters that are relevant to determine equilibrium effort, so we keep them constant to isolate the effect on equilibrium effort of aggregating the contests. We assume that an equilibrium exists in both the small and the large contests, which amounts to (2.8) with $m = n, kn$. Thus,

Proposition 1. *The effort of contestant i in the C contest is larger than in the D contest iff*

$$knY_i - n^2k + 1 > s. \tag{2.13}$$

Proof. We have that

$$\frac{\beta(kn-1+s)kV_i}{kY_i}\left(1 - \frac{kn-1}{kY_i}\right)$$
$$> \frac{\beta(n-1+s)V_i}{Y_i}\left(1 - \frac{n-1}{Y_i}\right) \Leftrightarrow \tag{2.14}$$
$$(kn-1+s)\left(1 - \frac{kn-1}{kY_i}\right)$$

[1] We assume that the value of the prize in C is just the sum of the k prizes in D.

[2] An identical assumption guarantees that when the CSF is of the Tullock type, all players are active in equilibrium, see Franke et al. (2013, Theorem 2.2).

[3] In other words, α is the only parameter that changes in order to maintain (2.3).