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Allocating value among farsighted players in network formation

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HIGHLIGHTS

- We study the stability of networks when players are farsighted.
- Allocations are determined endogenously.
- We propose the notion of von Neumann–Morgenstern farsighted stability with bargaining.
- Stability singles out the set of strongly efficient networks under some conditions.
- The componentwise egalitarian allocation rule emerges endogenously.

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ABSTRACT

We study the stability of networks when players are farsighted and allocations are determined endogenously. The set of strongly efficient networks is the unique von Neumann–Morgenstern farsightedly stable set with bargaining if the value function is anonymous, component additive and top convex and the allocation rule is anonymous and component efficient. Moreover, the componentwise egalitarian allocation rule emerges endogenously.

1. Introduction

We address the question of which networks one might expect to emerge in the long run when the players are farsighted and the allocation of value among players is determined together with the network formation. We propose the notion of von Neumann–Morgenstern (vNM) farsighted stability with bargaining. In contrast to Chwe's (1994) definition of vNM farsighted stability, allocations are going to be agreed upon among farsighted players when allocations and links are determined jointly. To capture this idea we request that the allocation rule satisfy the property of equal bargaining power for farsighted players. This property requires that, for each pair of players linked in the network, both players suffer or benefit equally from being linked with respect to their respective prospect. In addition, we request that each prospect can be reached by a farsighted improving path emanating from some network adjacent to the network over which bargaining takes place.[1]

We show that, if the value function is anonymous, component additive and top convex and the allocation rule is anonymous and component efficient, then the set of strongly efficient networks is the unique vNM farsightedly stable set with bargaining.

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[1] A farsighted improving path is a sequence of networks that can emerge when players form or sever links based on the improvement the end network offers relative to the current network.

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Moreover, the componentwise egalitarian allocation rule emerges endogenously.

Most papers that look at the endogenous determination of allocations together with network formation assume either simultaneous games with myopic players or sequential games with finite horizon and specific ordering (Currarini and Morelli, 2000; Mutuswami and Winter, 2002; Bloch and Jackson, 2007). More closely related to our work is Navarro (2014) who analyzes a dynamic process of network formation that is represented by means of a stationary transition probability matrix.[2] We rather adopt the stability approach because the noncooperative or dynamic approach is much sensitive to the specification of the bargaining game and network formation process, whose fine details (such as how the game ends) can be very important in determining what networks form and how value is allocated.

2. Networks, values and allocation rules

Let $N = \{1, \ldots, n\}$ be the finite set of players. A network g is a list of which pairs of player are linked to each other. We write $ij \in g$ to indicate that i and j are linked under the network g. Let g^S be the set of all subsets of $S \subseteq N$ of size 2. So, g^N is the complete network. The set of all possible networks on N is denoted by $\mathbb{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $g|_S = \{ij \mid ij \in g \text{ and } i \in S,\ j \in S\}$ be the network found deleting all links except those that are between players in S. Let $N(g) = \{i \mid \exists j \text{ such that } ij \in g\}$ be the set of players who have at least one link in the network g and let $N_i(g) = \{j \mid ij \in g\}$ be the neighborhood of player i. A network g' is adjacent to g if $g' = g + ij$ or $g' = g - ij$ for some ij. Let $A_i(g)$ be the set of adjacent networks to g deleting one of the link of player i. A path in a network $g \in \mathbb{G}$ between i and j is a sequence of players $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$, and such that each player in the sequence $i_1, \ldots, i_K$ is distinct. A non-empty network $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. The set of components of g is denoted by $C(g)$. Let $\Pi(g)$ denote the partition of N induced by the network g. That is, $S \in \Pi(g)$ if and only if either there exists $h \in C(g)$ such that $S = N(h)$ or there exists $i \notin N(g)$ such that $S = \{i\}$.

A value function is a function v that assigns a value $v(S, g)$ to every network g and every coalition $S \in \Pi(g)$. Given v, the total value that can be distributed at network g is equal to $v(g) = \sum_{S \in \Pi(g)} v(S, g)$. The set of all possible value functions v is denoted by $\mathcal{V}$. A value function v is *component additive* (Jackson and Wolinsky, 1996) if for any $g \in \mathbb{G}$ and $S \in \Pi(g)$, $v(S, g) = v(S, g|_S)$. Given a permutation of players π and any $g \in \mathbb{G}$, let $g^\pi = \{\pi(i)\pi(j) \mid ij \in g\}$. A value function v is *anonymous* (Jackson and Wolinsky, 1996) if for any permutation π, $g \in \mathbb{G}$ and $S \in \Pi(g)$, $v(\{\pi(i) \mid i \in S\}, g^\pi) = v(S, g)$. A network $g \in \mathbb{G}$ is *strongly efficient* relative to v if $v(g) \geq v(g')$ for any $g' \in \mathbb{G}$. Let $E(v)$ be the set of strongly efficient networks. Let $\rho(v, S) = \max_{g \subseteq g^S} v(g)/\#S$. A value function v is *top convex* (Jackson and van den Nouweland, 2005) if $\rho(v, N) \geq \rho(v, S)$ for any $S \subseteq N$.

An allocation rule y is a function that assigns a payoff $y_i(g, v)$ to player $i \in N$ from network g under the value function $v \in \mathcal{V}$. An allocation rule y is *component efficient* (Myerson, 1977) if for any $g \in \mathbb{G}$ and $S \in \Pi(g)$, $\sum_{i \in S} y_i(g, v) = v(S, g)$.[3] Given a permutation π, let v^π be defined by $v^\pi(S, g) = v(\{\pi^{-1}(i) \mid i \in S\}, g^{\pi^{-1}})$ for any $g \in \mathbb{G}$. An allocation rule y is *anonymous* (Jackson and Wolinsky, 1996) if for any v, $g \in \mathbb{G}$ and permutation π, $y_{\pi(i)}(g^\pi, v^\pi) = y_i(g, v)$. The egalitarian allocation rule y^e is defined by $y_i^e(g, v) = v(g)/n$. For a component additive v and network g, the componentwise egalitarian allocation rule y^{ce} is such that for any $S \in \Pi(g)$ and each $i \in S$, $y_i^{ce}(g, v) = v(S, g|_S)/\#S$. For a v that is not component additive, $y^{ce}(g, v) = v(g)/n$ for all g; thus, y^{ce} splits the value $v(g)$ equally among all players if v is not component additive.

3. vNM farsighted stability with bargaining

We first introduce the notions of farsighted improving path and prospect. A *farsighted improving path* (Jackson, 2008; Herings et al., 2009) from a network g to a network $g' \neq g$ is a finite sequence of graphs $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either: (i) $g_{k+1} = g_k - ij$ for some ij such that $y_i(g_K, v) > y_i(g_k, v)$ or $y_j(g_K, v) > y_j(g_k, v)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $y_i(g_K, v) > y_i(g_k, v)$ and $y_j(g_K, v) \geq y_j(g_k, v)$. Let $F(g)$ be the set of networks that can be reached by a farsighted improving path from g. A *prospect* z is a function that assigns to each network $g \in \mathbb{G}$ a network $z_i(g) \in \mathbb{G}$ for each player $i \in N$. Intuitively, when player i is bargaining how to share the surplus with other players she is linked to in g, she has in mind the payoff she might obtain at some other network, $z_i(g)$, not necessarily adjacent to g since players are farsighted.[4]

A set of networks is a vNM farsightedly stable set with bargaining if there exists an allocation rule and a prospect such that the following conditions hold. *Internal stability*: there is no farsighted improving path from one network inside the set to another network inside the set. *External stability*: from any network outside the set there is a farsighted improving path to some network inside the set. *Equal bargaining power*: the value of each network is allocated among players so that players suffer or benefit equally from being linked to each other compared to the allocation they would obtain at their respective prospect.[5] *Consistency*: the prospect can be reached by a farsighted improving path emanating from some network adjacent to the network over which bargaining takes place.

Definition 1. A set of networks $G \subseteq \mathbb{G}$ is a vNM farsightedly stable set with bargaining if there exists an allocation rule y and a prospect z such that

(i) $\forall g \in G,\ F(g) \cap G = \emptyset$; (Internal Stability)
(ii) $\forall g' \in \mathbb{G} \setminus G,\ F(g') \cap G \neq \emptyset$; (External Stability)
(iii) $\forall g \in \mathbb{G}$ and $ij \in g$,
 (a) $y_i(g, v) - y_i(z_i(g), v) = y_j(g, v) - y_j(z_j(g), v)$, (Equal Bargaining Power)

[2] Navarro (2014) shows that if players are quite impatient (or close to be myopic), then there exists an allocation rule together with a transition probability matrix such that the allocation rule is component efficient and the allocation rule together with the transition probability is an expected fair pairwise network formation procedure.

[3] An allocation rule y is *component balanced* (Jackson and Wolinsky, 1996) if for any component additive v, $g \in \mathbb{G}$ and $S \in \Pi(g)$, $\sum_{i \in S} y_i(g, v) = v(S, g|_S)$.

[4] Player i's prospect, $z_i(g)$, at network g, can be interpreted as her bargaining threat (what she expects to obtain in case an agreement is not reached; i.e. her payoff in the network she expects to end up) when she is negotiating the sharing of the surplus within her component. This prospect is endogenously determined.

[5] Equal bargaining power was originally defined for myopic players (see e.g. Myerson, 1977, Jackson and Wolinsky, 1996): for each link ij in g, both i and j should equally benefit or suffer taking as reference the adjacent network $g - ij$. Once players are farsighted, equal bargaining power requires that players equally benefit or suffer taking as reference network (or prospect), not necessarily adjacent networks, but networks that may be reached from adjacent networks through a sequence of networks when players form or delete links based on the improvement the end network offers relative to the current one.