Economics Letters 130 (2015) 1–4

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A model-free test for contagion between crude oil and stock markets☆

Zhiyuan Pan*, Xu Zheng, Yuting Gong

Antai College of Economics & Management, Shanghai Jiao Tong University, China

HIGHLIGHTS

- This paper extends Hong et al. (2007)'s test for contagion analysis.
- Our test has reasonable size and good power in finite sample.
- We apply this test to crude oil and stock return data.
- Empirical results reveal the strong evidence of contagion.

ARTICLE INFO

Article history:
Received 3 March 2014
Received in revised form
14 February 2015
Accepted 16 February 2015
Available online 25 February 2015

JEL classification:
C12
G1

Keywords:
Contagion
Model-free test
Crude oil market
Stock market

ABSTRACT

This paper extends Hong et al. (2007)'s model-free test to analyze the contagion. A simulation experiment reveals that our test has reasonable size and good power in finite sample. We use this test and find the strong evidence of contagion between crude oil and stock markets.

1. Introduction

Since the influential paper of Forbes and Rigobon (2002), the contagion among different markets has been of great interest to academics.[1] There are many models that have been employed to examine contagion, such as extreme value model (Longin and Solnik, 2001), bivariate GARCH (Chiang et al., 2007) and the family of Copula models (Wen et al., 2012). These models above do indeed have some advantages, but they are inappropriate if the assumptions of correct model specifications do not hold (Wang et al., 2015). Specifically, the normal assumption implied bivariate GARCH is not suitable for financial time series with non-normal features (skewness and leptokurtosis). Extreme value model is criticized for its definition of extreme observations (Rodriguez, 2007).

In this study, we contribute to the literature by proposing a model-free test for detecting contagion. This is actually an extension of the exceedance correlation test of Hong et al. (2007). A simulation experiment reveals that the test has reasonable size and good power in finite sample. We also apply this new test to examine the contagion between crude oil and stock markets during the financial crisis in 2007–2008.

2. Methodology

Let $R_{1,t}$ and $R_{2,t}$ be random variables with zero mean and unit variance. Following Hong et al. (2007) and many others, we define the conditional correlation given the pre-determined threshold value T_c as

$$\rho^{+}(T_c) = corr(R_{1,t}, R_{2,t} | t > T_c), \quad (1)$$

$$\rho^{-}(T_c) = corr(R_{1,t}, R_{2,t} | t < T_c). \quad (2)$$

☆ This work was supported by the National Natural Science Foundation of China (No. 70971087).

* Correspondence to: 535 Fahua Zhen Rd., Shanghai 200052, China. Tel.: +86 15021412552.
E-mail address: panzhiyuan2012@gmail.com (Z. Pan).

[1] Forbes and Rigobon (2002) define contagion as a significant increase in cross-market linkages after a shock.

http://dx.doi.org/10.1016/j.econlet.2015.02.023
0165-1765/

Notice that the subperiods are separated by time point T_c, rather than by both returns being above or below an exceedance level c in the most literatures[2] (Longin and Solnik, 2001; Ang and Chen, 2002; Hong et al., 2007; Pan et al., 2014). This modification in our paper is to investigate the difference in the cross-market correlations before and after the financial crisis. Thus, the null hypothesis of no contagion is

$$H_0 : \rho^+(T_c) = \rho^-(T_c), \tag{3}$$

and the alternative hypothesis is

$$H_1 : \rho^+(T_c) \neq \rho^-(T_c). \tag{4}$$

We are interested in testing whether the cross-market correlation changes significantly after a shock. If Eq. (3) is rejected, there must exist significantly different correlation.

To formulate a feasible statistic, we first estimate the conditional correlation in Eqs. (1)–(2) utilizing their sample analogues. The sample conditional mean and variance are given as

$$\hat{\mu}_i^+(T_c) = \frac{1}{N_c^+}\sum_{t=1}^{T} R_{i,t}\mathbf{1}(t > T_c), \tag{5}$$

$$\hat{\sigma}_i^+(T_c)^2 = \frac{1}{N_c^+ - 1}\sum_{t=1}^{T}[R_{i,t} - \hat{\mu}_i(T_c)]^2\mathbf{1}(t > T_c), \tag{6}$$

where $i \in \{1, 2\}$; N_c^+ is the number of observations when time t is large than T_c and T is sample size; $\mathbf{1}(\cdot)$ denotes the indicator function. Therefore, the sample conditional correlation $\hat{\rho}^+(T_c)$ can be given as

$$\hat{\rho}^+(T_c) = \frac{1}{N_c^+}\sum_{t=1}^{T}\hat{X}_{1,t}^+(T_c)\hat{X}_{2,t}^+(T_c)\mathbf{1}(t > T_c), \tag{7}$$

where

$$\hat{X}_{1,t}^+(T_c) = \frac{R_{1,t} - \hat{\mu}_1^+(T_c)}{\hat{\sigma}_1^+(T_c)}, \qquad \hat{X}_{2,t}^+(T_c) = \frac{R_{2,t} - \hat{\mu}_2^+(T_c)}{\hat{\sigma}_2^+(T_c)}. \tag{8}$$

Similarly, $\hat{\rho}^-(T_c)$ can be computed as $\hat{\rho}^+(T_c)$.

Then, the test statistic can be constructed as

$$J = T[\hat{\rho}^+(T_c) - \hat{\rho}^-(T_c)]\hat{\Omega}^{-1}[\hat{\rho}^+(T_c) - \hat{\rho}^-(T_c)], \tag{9}$$

where $\hat{\Omega}$ is a consistent estimator of the variance for $\sqrt{T}[\hat{\rho}^+(T_c) - \hat{\rho}^-(T_c)]$, which is given by

$$\hat{\Omega} = \sum_{l=1-T}^{T-1} k(l/p)\hat{\gamma}_l \tag{10}$$

where the smoothing parameter p is chosen as 3 similar to Hong et al. (2007) and $k(\cdot)$ is the Bartlett kernel function,

$$k(z) = (1 - |z|)\mathbf{1}(|z| < 1), \tag{11}$$

and $\hat{\gamma}_l$ is

$$\hat{\gamma}_l = \frac{1}{T}\sum_{t=|l|+1}^{T}\hat{\xi}_t(T_c)\hat{\xi}_{t-|l|}(T_c), \tag{12}$$

with

$$\begin{aligned}\hat{\xi}_t(T_c) = {} & \frac{T}{N_c^+}[\hat{X}_{1,t}^+(T_c)\hat{X}_{2,t}^+(T_c) - \hat{\rho}^+(T_c)]\mathbf{1}(t > T_c) \\ & - \frac{T}{N_c^-}[\hat{X}_{1,t}^-(T_c)\hat{X}_{2,t}^-(T_c) - \hat{\rho}^-(T_c)]\mathbf{1}(t \leq T_c).\end{aligned} \tag{13}$$

The following theorem gives the asymptotic property for statistic J in Eq. (9):

Theorem 1. *Under the null hypothesis H_0 and Assumptions 1–4, as the sample size T is sufficiently large,*

$$J \to^d \chi^2(1). \tag{14}$$

Proof of Theorem 1 is given in the Appendix.

3. A Monte Carlo experiment

To investigate the finite sample performance of the test J, we conduct a Monte Carlo experiment to examine the size and power of the test.

We generate two return series using following model specification which is also used in Engle (2002):

$$r_{1,t} = \sqrt{h_{1,t}}\epsilon_{1,t}, \qquad r_{2,t} = \sqrt{h_{2,t}}\epsilon_{2,t}, \tag{15}$$

$$h_{1,t} = 0.01 + 0.05r_{1,t-1}^2 + 0.94h_{1,t-1}, \tag{16}$$

$$h_{2,t} = 0.5 + 0.2r_{2,t-1}^2 + 0.5h_{2,t-1} \tag{17}$$

$$\begin{pmatrix}\epsilon_{1,t}\\ \epsilon_{2,t}\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}0\\0\end{pmatrix} \quad \begin{pmatrix}1 & \rho_t\\ \rho_t & 1\end{pmatrix}\right]. \tag{18}$$

The correlation process is set as:

$$\rho_t = 0.15 + \lambda \times \mathbf{1}(t \geq T/2) \tag{19}$$

where $\mathbf{1}(\cdot)$ is an indicator function and T denotes the sample size. Obviously, $\lambda \in (-1.15, 0.85)$[3] is the threshold parameter. That is, if λ equals to 0, then the correlations are the same in the whole sample. For this scenario, we can observe the performance of the size of the test. If λ deviates from 0, then the correlations are divided into two parts, 0.15 and $0.15 + \lambda$. For this case, we can examine the behavior of the power of the test. Therefore, $\lambda = -0.6$, -0.3, 0, 0.3, 0.6 are chosen in our simulation.

Sample of $T = (250, 500, 750, 1000)$ observations are investigated, corresponding to 1 year, 2 years, 3 years and 4 years for a market that has 250 business days per year. The sample begins to be counted after the first 2000 observations are discarded to avoid any effect from starting values for the parameters. In all cases, the results are based on 1000 Monte Carlo simulations.

Panel A in Table 1 reports the size of the test J at 1%, 5% and 10% significance level, respectively. We can see that the test has reasonable size in most cases. The sizes are close to the corresponding nominal sizes, which implies that our test seems quite reliable under the null.

To evaluate power, λ is set to be different from 0. In this paper, we let λ equal to -0.6, -0.3, 0.3, 0.6. Clearly, the farther from zero it is, the more it deviates from the same correlations to have more contagion. Panels B–E in Table 1 show that the proportion of rejections increases with sample size, as expected, and quickly converges to one when λ is farther from zero. The findings illustrate that our test has good power in finite sample.

4. Empirical application

In this section, we will use our new test to examine the contagion between crude oil and stock markets. We use the spot price data of WTI and Brent crude oil which are the benchmarks of world oil pricing. We choose three popular stock indices, that is S&P 500 index (US), FTSE 100 (UK) and the DAX index (Germany).[4] To avoid the non-synchronous trading, we remove those observations

[2] Take ρ^+ for example, $\rho^+(c)$ is defined as $corr(R_{1,t}, R_{2,t}|R_{1,t} > c, R_{2,t} > c)$ in the asymmetry literature.

[3] This guarantees the correlation in a interval $(-1, 1)$ at all times.

[4] The crude oil spot prices are obtained from http://www.eia.gov, and stock indices are available from http://finance.yahoo.com.