



Testing for joint significance in nonstationary ordered choice model



Peng Xu*

School of Economics, Nankai University, China
Capital Markets CRC, Australia

HIGHLIGHTS

- We investigate the joint significance test for nonstationary ordered choice model.
- The convergence rate of maximum likelihood estimator of explanatory variable is T .
- Wald statistic is asymptotically Chi-squared for nonstationary logit or probit model.

ARTICLE INFO

Article history:

Received 23 September 2014
Received in revised form
5 February 2015
Accepted 13 February 2015
Available online 24 February 2015

JEL classification:

C12
C25

Keywords:

Ordered choice model
Nonstationarity
Significance test

ABSTRACT

This paper studies the test of joint significance for the ordered choice model with multiple explanatory variables following integrated processes. The results show that for the widely used logit and probit models, when the true parameter vector of explanatory variables is zero, the classical Wald statistic is still useful and has a Chi-squared limiting distribution.

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1. Introduction

Nonstationary ordered choice models have attracted lots of attention in the process of the financial and macroeconomic time series modeling. Hu and Phillips (2004), Phillips et al. (2007) have developed an asymptotic theory for time series ordered choice models with explanatory variables generated as integrated processes. Their theory fundamentally depends on the assumption that $\|\beta_0\| \neq 0$, where β_0 is the true parameter vector of explanatory variables and $\|\cdot\|$ is the Euclidean norm. In practice, however, we may not have prior information about this assumption. Therefore, it should test $\|\beta_0\| = 0$ before applying their results.

In this paper, we investigate the joint significance test for the ordered choice model with multiple integrated explanatory

variables. The paper is structured as follows. Section 2 outlines the model and assumptions. Section 3 gives the main results of the limiting distribution of the Wald statistic and also presents some Monte Carlo simulation results. Some useful proofs of the main theorems are given in the Appendix of Xu (2015).

2. Model and assumptions

The nonstationary ordered choice model is defined as

$$y_t^* = x_t' \beta_0 - \varepsilon_t \quad \text{for } t = 1, \dots, T \quad (1)$$

where x_t is a k -dimensional vector of explanatory variables and ε_t is an error term which is independently and identically distributed (*i.i.d.*) with cumulative distribution function F . The dependent variable y_t^* is assumed to be unobserved and can be indicated by y_t

* Correspondence to: 3/55 Harrington Street, Sydney, NSW, 2000, Australia. Tel.: +61 0481285669.

E-mail address: masxupeng@126.com.

$$\begin{aligned}
y_t &= 0 \quad \text{if } y_t^* \in (-\infty, \mu_0^1] \\
&= 1 \quad \text{if } y_t^* \in (\mu_0^1, \mu_0^2] \\
&= 2 \quad \text{if } y_t^* \in (\mu_0^2, \mu_0^3] \\
&\vdots \\
&= J-1 \quad \text{if } y_t^* \in (\mu_0^{J-1}, \mu_0^J] \\
&= J \quad \text{if } y_t^* \in (\mu_0^J, \infty).
\end{aligned} \tag{2}$$

Note that the threshold parameters in (2) are not scaled, which is different from Hu and Phillips (2004), because the thresholds have the same order of probability as ε_t in (1) when $\|\beta_0\| = 0$. To derive the limiting distribution results, we make the following assumptions regarding the data generating processes x_t .

Assumption 1. Let x_t follow a k -dimensional integrated processes,

$$x_t = x_{t-1} + v_t$$

where $x_0 = 0$ and $v_t \sim i.i.d.(0, \Omega_v)$. By Functional Central Limit Theorem we have

$$T^{-1/2}x_{[Tr]} \Rightarrow B_v(r)$$

where $[\cdot]$ denotes the floor function, $\Rightarrow$ signifies weak convergence, and $B_v(r)$ is the Brownian motion with positive definite covariance matrix Ω_v .

In the ordered discrete choice model, the conditional probability distribution of y_t is given by

$$\begin{aligned}
P_0(x_t; \theta_0) &= 1 - F(x'_t \beta_0 - \mu_0^1) \\
P_j(x_t; \theta_0) &= F(x'_t \beta_0 - \mu_0^j) - F(x'_t \beta_0 - \mu_0^{j+1}) \\
&\quad \text{for } j = 1, \dots, J-1 \\
P_J(x_t; \theta_0) &= F(x'_t \beta_0 - \mu_0^J)
\end{aligned}$$

where $P_j(x_t; \theta_0) = P(y_t = j | \mathcal{F}_{t-1})$, $\theta_0 = (\beta_0', \mu_0')'$, $\mathcal{F}_{t-1}$ is some filtration. Then we have conditional expectation of y_t

$$m(x_t; \theta_0) = \sum_{j=1}^J j \cdot P_j(x_t; \theta_0).$$

Define $u_t = y_t - m_t$, it is apparent that $(u_t, \mathcal{F}_t)$ is a martingale difference with conditional moments

$$\sigma_k(x_t; \theta_0) = E(u_t^k | \mathcal{F}_{t-1}).$$

Define $z_{kt} = u_t^k - \sigma_{kt}$, $\eta_{klt} = E(z_{kt} z_{lt} | \mathcal{F}_{t-1})$ and $a_{klt} = z_{kt} z_{lt} - \eta_{klt}$. Then we can see that $(z_{kt}, \mathcal{F}_t)$ and $(a_{klt}, \mathcal{F}_t)$ are also martingale difference sequence. Furthermore, we define the fourth conditional expectation of z_{kt} as $\tau_{klmnt} = E(a_{klt} a_{mnt} | \mathcal{F}_{t-1})$. These moment conditions are very useful in proving Theorems 1 and 2.

Let the indicator function be

$$\Lambda(t, j) = \frac{\prod_{i=0, \dots, J \& i \neq j} (y_t - i)}{\prod_{i=0, \dots, J \& i \neq j} (j - i)}.$$

Then the log likelihood function has the form

$$\log L_T(\theta) = \sum_{t=1}^T \sum_{j=0}^J \Lambda(t, j) \log P_j(x_t; \theta).$$

Let the first and second derivatives of F be denoted as f and $\dot{f}$. The elements of the score function $S_T(\theta) = (S_T(\beta)', S_T(\mu)')' = (\partial \log L_T / \partial \beta', \partial \log L_T / \partial \mu')'$ are

$$\frac{\partial \log L_T}{\partial \beta} = \sum_{t=1}^T \sum_{j=0}^J \frac{\Lambda(t, j)}{P_j(x_t; \theta)} p_j(x_t; \theta) x_t \tag{3}$$

$$\frac{\partial \log L_T}{\partial \mu^j} = \sum_{t=1}^T \left(\frac{\Lambda(t, j-1)}{P_{j-1}(x_t; \theta)} - \frac{\Lambda(t, j)}{P_j(x_t; \theta)} \right) f(x'_t \beta - \mu^j) \tag{4}$$

where

$$\begin{aligned}
p_0(x_t; \theta) &= -f(x'_t \beta - \mu^1) \\
p_j(x_t; \theta) &= f(x'_t \beta - \mu^j) - f(x'_t \beta - \mu^{j+1}) \quad \text{for } j = 1, \dots, J-1 \\
p_J(x_t; \theta) &= f(x'_t \beta - \mu^J).
\end{aligned}$$

Since the expected value of the ratio $\Lambda(t, j)/P_j(x_t; \theta)$ is 1, the ratio can be written as a sum of martingale differences

$$\Lambda(t, j) = \sum_{k=1}^J g_k(x_t; j, \theta_0) z_{kt} + 1 \tag{5}$$

where $g_k(j)$ is the coefficient of z_{kt} for a given j . Using (5), we rewrite (3) and (4) as

$$\frac{\partial \log L_T}{\partial \beta} = \sum_{t=1}^T \sum_{k=1}^J A_k(x_t; \theta) z_k(x_t; \theta) x_t$$

$$\frac{\partial \log L_T}{\partial \mu^j} = \sum_{t=1}^T \sum_{k=1}^J B_k(x_t; j, \theta) z_k(x_t; \theta)$$

where

$$\begin{aligned}
A_k(x_t; \theta) &= \sum_{j=0}^J g_k(x_t; j, \theta) p_j(x_t; \theta) \\
&= \sum_{j=1}^J f(x'_t \beta - \mu^{j+1}) [g_k(x_t; j, \theta) - g_k(x_t; j-1, \theta)] \tag{6}
\end{aligned}$$

$$B_k(x_t; j, \theta) = (g_k(x_t; j-1, \theta) - g_k(x_t; j, \theta)) f(x'_t \beta - \mu^{j+1}).$$

Since the parameters are commonly expressed in two types, we write the Hessian matrix $H_T(\theta)$ as

$$H_T(\theta) = \begin{pmatrix} H_{T,11}(\theta) & H_{T,12}(\theta) \\ H_{T,21}(\theta) & H_{T,22}(\theta) \end{pmatrix}. \tag{7}$$

The detailed forms of $H_T(\theta)$ are shown in Appendix A of Xu (2015). Since the ML estimator involves nonlinear functions of the process x_t , it helps to make some assumptions about the distribution function F of ε_t .

Assumption 2. F is three times differentiable. Further, for $k, l = 1, \dots, J$

- (a) $A_k A_l \eta_{kl}$, $A_k B_l \eta_{kl}$, $B_k B_l \eta_{kl}$ are bounded.
- (b) $(A_k \dot{B}_l z_k z_l)$, $(A_k \dot{A}_l z_k z_l)$, $(B_k \dot{B}_l z_k z_l)$, $(\dot{C}_k z_k)$ are bounded.
- (c) $\tau_{klpq} A_k A_l A_p A_q$, $\tau_{klpq} A_k A_l B_p B_q$, $\tau_{klpq} B_k B_l B_p B_q$ are bounded.

Following similar techniques as in Park and Phillips (2000), it can be shown that both probit and logit models satisfy Assumption 2.

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