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Residual-based test for fractional cointegration

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HIGHLIGHTS

- We propose a residual-based test for fractional cointegration.
- The integration orders can be real-valued and the resulting cointegrating error can be stationary or nonstationary.
- The proposed test is simple to implement, has standard asymptotics and does not require a prescribed bandwidth.
- The proposed test has better power than the GPH test for unit-root series and has satisfactory sizes when other tests fail.

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ABSTRACT

By allowing deviations from equilibrium to follow a fractionally integrated process, the notion of fractional cointegration analysis encompasses a wide range of mean-reverting behaviors. For fractional cointegrations, asymptotic theories have been extensively studied, and numerous empirical studies have been conducted in finance and economics. But as far as testing for fractional cointegration is concerned, most of the testing procedures have restrictions on the integration orders of observed time series or integrating error and some tests involve determination of bandwidth. In this paper, a general fractional cointegration model with the observed series and the cointegrating error being fractional processes is considered, and a residual-based testing procedure for fractional cointegration is proposed. Under some regularity conditions, the test statistic has an asymptotic standard normal distribution under the null hypothesis of no fractional cointegration and diverges under the alternatives. This test procedure is easy to implement and works well in finite samples, as reported in a Monte Carlo experiment.

1. Introduction

In the past two decades, the concept of fractional cointegration, which allows the equilibrium error to follow a fractional integrated process, has received much attention in the finance and econometric literature. A partial list of some of the recent developments in this area includes Cheung and Lai (1993) and Soofi (1998) who test the purchasing power parity hypothesis, Baillie and Bollerslev (1994) and Hassler et al. (2006), who investigate the memory of exchange rates, and Booth and Tse (1995) and Dittmann (2000) who explore the dynamic of interest rate future markets and stock market prices, respectively. All of these studies detect evidence of fractional cointegration and obtain satisfactory result under the assumption that the observations are $I(1)$ processes.

Testing for fractional cointegration has subsequently been generalized to fractionally integrated processes. Robinson (2008) proposes a test based on the joint local Whittle estimation of all parameters, which rules out the possibility that the two underlying series have equal integration orders. For time series with equal integration orders, Marinucci and Robinson (2001) propose a Hausman-type test for no cointegration, which involves determination of a bandwidth. Marmol and Velasco (2004) construct a Hausman-type test to test for the presence of fractional cointegration, with the additional assumption that the cointegration error is nonstationary.

In this paper, we derive a general cointegration testing procedure for two fractionally integrated processes with equal integration orders, which are assumed to be unknown. This test allows the cointegrating error to be fractionally integrated without requiring them to be nonstationary. The test is shown to be asymptotically standard normal under the null hypothesis of no fractional cointegration and diverges under the fractional cointegration alternative.

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The test is residual-based, which was initially suggested by Engle and Granger (1987) and studied by Phillips and Ouliaris (1990). The methodology does not require further specification of the short-run dynamics of the underlying processes because semiparametric estimates of the long-memory parameters and the long-run covariance matrix are used. Only preliminary estimates of the integration orders of the underlying series and the regression residual are needed. This test is easy to implement and does not require any user-chosen number such as bandwidth.

The proposed test requires the integration orders of the observed series to be equal, which can be determined by the residual-based test proposed by Hualde (2013) or Wang (2008), both of which are valid irrespective of whether the series are cointegrated.

In the next section, we present the testing procedure and the asymptotic theory. Section 3 reports the empirical sizes and powers of our test via a Monte Carlo study. Section 4 concludes.

2. Testing for fractional cointegration

Consider the following bivariate model $(y_t, x_t)'$, with prime denoting transposition and $t \in \{0, \pm 1, \pm 2, \ldots\}$,

$$y_t = \beta x_t + \Delta^{-\delta}\{\vartheta_{1t} 1(t > 0)\},$$
$$x_t = \Delta^{-d}\{\vartheta_{2t} 1(t > 0)\}, \quad (1)$$

where $1(\cdot)$ is the indicator function, $\delta = 1 - L$, L is the lag operator, $d > 1/2$, $\delta \le d$, and $\upsilon_t = (\upsilon_{1t}, \upsilon_{2t})'$ is white noise defined in Assumption 1 below. We concentrate on the case that x_t and y_t are both nonstationary with $d > 1/2$. The case $\delta < d$ indicates the existence of fractional cointegration. By Taylor's expansion, $\Delta^{-d} = \sum_{j=0}^{\infty} \pi_j(-d) L^j$, $\pi_j(d) = \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)}$ for $d \ne 0, -1, -2, \ldots$, where $\Gamma(\cdot)$ is the gamma function, taking $\Gamma(\alpha) = \infty$ for $\alpha = 0, -1, -2, \ldots$, $\Gamma(0)/\Gamma(0) = 1$. Denote the $k \times k$ identity matrix by I_k and the Euclidean norm by $\|\cdot\|$.

Assumption 1. Consider the process $\upsilon_t = A(L)\epsilon_t$, $A(L) = \sum_{j=0}^{\infty} A_j L^j$. Assume that

1.1. $\det(A(1)) \ne 0$ and $\sum_{j=1}^{\infty} j \|A_j\|^2 < \infty$;
1.2. ϵ_t are i.i.d. vectors with mean zero, positive definite covariance matrix Ω, and $E\|\epsilon_t\|^q < \infty$, for some $q > \max(4, 2/(2d-1))$;
1.3. $f_{ii}(0) > 0$, $i = 1, 2$, where $f_{ij}(0)$ is the (i, j) element of the spectral density of υ_t, denoted by $f(\lambda)$.

Assumption 1 is common because it is satisfied by the usual stationary and invertible autoregressive moving average (ARMA) processes. This assumption is similar to Assumption 1 of Robinson and Hualde (2003), and Assumptions A–C of Marmol and Velasco (2004), which is a condition for applying the functional limit theorem of Marinucci and Robinson (2000). Assumption 1.1 ensures that the limiting process of partial sum of υ_t has nondegenerate finite-dimensional distributions. Assumption 1.2 along with Assumption 1.3 imply that $f(\lambda)$ is $Lip(\gamma)$, $\gamma > 0$, which enables one to obtain the asymptotic properties of the test statistic, see Theorem 1 below. A larger d entails weaker moment conditions. If $d > 3/4$, $\max(4, 2/(2d-1)) = 4$, then d has no restriction on the moments of ϵ_t.

Under Assumption 1, model (1) means that x_t and y_t are both type-II fractionally integrated $I(d)$ processes and a linear combination $y_t - \beta x_t$ is $I(\delta)$. In this way, the traditional $I(1)$ cointegration is a special case of model (1) with $d = 1$ and $\delta = 0$.

Denote the regression residual in (1) by $u_t = y_t - \beta x_t$, then u_t is an $I(\delta)$ process. Therefore, testing the hypothesis of no fractional cointegration between y_t and x_t against existence of fractional cointegration can be formulated as $H_0 : \delta = d$ against $H_1 : \delta < d$.

To construct the test statistic, it is important to estimate δ and d precisely, thus we impose the following assumption.

Assumption 2. Under both the null and the alternative hypothesis,

2.1. there exists a positive constant $K < \infty$ and estimates $\hat{d}$, $\hat{\delta}$ of d and δ respectively, such that

$$|\hat{d}| + |\hat{\delta}| \le K, \quad (2)$$

and for some $\eta > 0$,

$$\hat{d} = d + O_p(T^{-\eta}), \quad (3)$$

$$\hat{\delta} = \delta + O_p(T^{-\eta}); \quad (4)$$

2.2. $\hat{f}(0) \xrightarrow{p} f(0)$, where $\xrightarrow{p}$ stands for convergence in probability.

Assumption 2.1 is the same as Assumption 3 of Robinson and Hualde (2003) and Assumption 2 of Hualde and Velasco (2008). Condition (2) is not restrictive if our estimates are optimizers of the corresponding functions over compact sets. The parameter d can be estimated from x_t by parametric or semiparametric memory estimates, for example the approximate Gaussian maximum likelihood estimates proposed by Beran (1995) or Whittle pseudo-maximum likelihood estimation proposed by Velasco and Robinson (2000), thus condition (3) is easily satisfied.

Condition (4) is more subtle because β is unknown and hence u_t is unobserved. The estimate of δ requires a proxy $\hat{u}_t$, which is an estimate of u_t. Let $\hat{\beta}$ be the Ordinary Least Squares (OLS) or Narrow Band (NB, see Robinson and Marinucci (2001)) estimates of β. Then under Assumptions 1.2 and 1.3, and some other mild conditions, estimation of the memory parameter of residuals $\hat{u}_t = y_t - \hat{\beta} x_t$ leads to a consistent estimate of δ under the null and the alternative hypotheses (cf. Velasco (2003), Hualde and Velasco (2008)).

Let $\hat{F} = F(\hat{\delta}, \hat{f}_{22}(0)) = \frac{T^{-1/2}\sum_t \Delta^{\hat{\delta}} x_t}{(2\pi \hat{f}_{22}(0))^{1/2}}$ be the test for $H_0 : \delta = d$ against the alternative $H_1 : \delta < d$.

Theorem 1. *Let Assumptions 1 and 2 hold, x_t and y_t are defined in (1), then $\hat{F} \xrightarrow{d} N(0, 1)$ under H_0 and $\hat{F} = O_p(T^{d-\delta})$ under H_1, where $\xrightarrow{d}$ stands for convergence in distribution.*

Proof. Since $\Delta^d x_t = \upsilon_{2t}$, which is an $I(0)$ process, it follows that $F(\delta, f_{22}(0)) = \frac{T^{-1/2}\sum_t \Delta^{-(d-\delta)}\upsilon_{2t}}{(2\pi f_{22}(0))^{1/2}}$. Under H_0, $F(\delta, f_{22}(0)) = \frac{T^{-1/2}\sum_t \upsilon_{2t}}{(2\pi f_{22}(0))^{1/2}}$, then $F(\delta, f_{22}(0)) \xrightarrow{d} N(0, 1)$ in view of the functional limit theory of $I(0)$ process. Under Assumptions 1 and 2, $\hat{\delta}$ and $\hat{f}_{22}(0)$ are consistent estimates of δ and $f_{22}(0)$. It can be proved that $F(\delta, f_{22}(0)) - \hat{F} = o_p(1)$ (see Appendix). Consequently $\hat{F} \xrightarrow{d} N(0, 1)$. Under H_1, $\Delta^{\delta} x_t$ is $I(d-\delta)$ since x_t is $I(d)$ process, then $T^{-1/2}\sum_{t=1}^{T} \Delta^{-\delta} x_t = O_p(T^{d-\delta})$, and further $F(\delta, f_{22}(0)) = O_p(T^{d-\delta})$. Since $\hat{\delta}$ and $\hat{f}_{22}(0)$ are consistent estimates of δ and $f_{22}(0)$, with Assumptions 1 and 2, it can be proved that $F(\delta, f_{22}(0)) - \hat{F} = o_p(T^{d-\delta})$ (see Appendix), thus $\hat{F} = O_p(T^{d-\delta})$. □

3. Monte Carlo simulations

Monte Carlo experiments are conducted to examine the finite sample performance of the test. Let $(y_t, x_t)'$ be generated from model (1) with $\beta = 1$, $\upsilon_t = (\upsilon_{1t}, \upsilon_{2t})'$ being a Gaussian white noise with $E(\upsilon_t) = \mathbf{0}$, $Var(\upsilon_{1t}) = Var(\upsilon_{2t}) = 1$ and $Cov(\upsilon_{1t}, \upsilon_{2t}) = \rho$. The initial values $\upsilon_{1t}, \upsilon_{2t}, t \le 0$ are set to be zero. We consider cases with $\rho = 0$ and 0.5, and sample sizes $T = 100$, 250, and 500. Let $d = 0.6, 0.8, 1, 1.2$ and for a given d, let $\delta = d, d-0.2, d-0.4$ and $d-0.6$. For a given set of (ρ, d, δ, T) and $\beta = 1$, we obtain the observed series $y_t, x_t, t = 1, \ldots, T$, and further testing procedure