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Variance change-point detection in panel data models

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HIGHLIGHTS

- We extend the CUSUM-based statistic in Horváth and Hušková (2012) to the variance shift panel data models.
- Asymptotic distribution is derived under the null hypothesis and the consistency of the test is proven under the alternative.
- We provide Monte Carlo evidence of the good small sample performance of this statistic.

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ABSTRACT

This paper proposes a cumulative sum (CUSUM) based statistic to test if there is a common variance change-point in panel data models. Asymptotic distribution is derived under the null hypothesis and the consistency of the test is proven under the alternative hypothesis. Monte Carlo experiment is carried out to show the effectiveness of the proposed procedure.

1. Introduction

Structural change problem has attracted much attention in statistics and econometrics. Important issues arising from this problem include detecting the change points, testing the number of changes and measuring the magnitude of changes. For surveys we refer to Csörgö and Horváth (1997), Leybourne and Taylor (2004), Perron (2006), Kramer and Kampen (2011) and Chen and Tian (2014). Recently there has been a growing literature on the estimation and tests of common breaks in panel data models in which there are N individual units and T time series observations for each individual. Joseph and Wolfson (1992) initiated change point models for panel data. Im et al. (2005) and Bai and Carrion-i-Silvestre (2009) discuss the analysis of panel data with possible change points in case of stationary and non-stationary errors. Feng et al. (2010) study the estimation of a single change point in panel models via a Wald-type statistic and Baltagi et al. (2012) extend it to allow for nonstationary regressors and innovations. Bai (2010) develops the asymptotic properties of least-squares and quasi-maximum likelihood change point estimators in panel location models. Horváth and Hušková (2012) propose a CUSUM-based test for the mean of panel data models.

In this paper, we propose a CUSUM-based test for the variance of panel data models, and the priori that there exists a variance change that is not needed. The asymptotic distribution of the test statistic is derived under the no change null hypothesis and consistency of the test is proven under the alternative hypothesis. Monte Carlo simulation results confirm the validity of the proposed method.

2. Models and assumptions

Consider a panel data model in which there are N panels and each panel has T observations,

$$X_{i,t} = \mu_i + e_{i,t}, \quad i = 1, \ldots, N, \ t = 1, \ldots, T, \tag{1}$$

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where

$$e_{i,t} = \sum_{l=0}^{\infty} c_{i,l}\varepsilon_{i,t-l}, \quad i = 1, \ldots, N,\ t = 1, \ldots, T, \tag{2}$$

$$E\varepsilon_{i,0} = 0, \qquad E\varepsilon_{i,0}^2 = 1, \qquad E\left|\varepsilon_{i,0}\right|^8 < \infty \quad \text{and}$$
$$\limsup_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} E\left|\varepsilon_{i,0}\right|^8 < \infty, \tag{3}$$

the sequences $\left\{\varepsilon_{i,t}, -\infty < t < \infty\right\}$ are independent of each other. (4)

for every i the variables $\left\{\varepsilon_{i,t}, -\infty < t < \infty\right\}$ are i.i.d. (5)

$\left|c_{i,l}\right| \le c_0 (l+1)^{-\alpha}$ for all $1 \le i \le N,\ 0 \le l < \infty,$ with some c_0 and $\alpha > 2.5,$ (6)

and

there is $\delta > 0$ such that $a_i^2 \ge \delta^2$ with $a_i = \sum_{l=0}^{\infty} c_{i,l}$ for all $1 \le i \le N.$ (7)

Since

$$\lim_{T\to\infty} \frac{1}{T} E\left(\sum_{t=1}^{T} e_{i,t}\right)^2 = \sigma_i^2, \quad 1 \le i \le N, \tag{8}$$

we obtain immediately from assumptions (3) and (7) that $a_i^2 = \sigma_i^2$ and

$$\sigma_i^2 \ge \delta^2 \quad \text{for all } 1 \le i \le N. \tag{9}$$

In this article, we wish to test the following hypothesis:

H_0 : the variance of panel i will not change for all $1 \le i \le N$ during the observation period,

H_1 : the variance of panel i changes from σ_i^2 to $\sigma_i^2 + \delta_i$ for all $1 \le i \le N$at time t_0,

where $\delta_i > 0$, t_0 is unknown, $t_0 = \lfloor T\tau_0 \rfloor$, $\tau_0 \in (0, 1)$, $\lfloor\cdot\rfloor$ denotes the integer part.

3. Main results

Let

$$V_{N,T}(x) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left\{\frac{1}{\hat{\varphi}_i}\frac{1}{\sqrt{T}}\left(\sum_{t=1}^{\lfloor Tx\rfloor}\hat{e}_{i,t}^2 - \frac{\lfloor Tx\rfloor}{T}\sum_{t=1}^{T}\hat{e}_{i,t}^2\right)\right\}, \quad 0 \le x \le 1, \tag{10}$$

be the CUSUM-based test statistic, if for any i, the errors $\left\{e_{i,t}, 1 \le t \le T\right\}$ are i.i.d.,

$$\hat{\varphi}_i^2 = T^{-1}\sum_{t=1}^{T}\hat{e}_{i,t}^4 - \left(T^{-1}\sum_{t=1}^{T}\hat{e}_{i,t}^2\right)^2, \quad i = 1, \ldots, N,$$

if independence cannot be assumed,

$$\hat{\varphi}_i^2 = \sum_{|h|\le h_T}\hat{\gamma}_i(h), \quad h_T = O(T^{\rho}), \ \rho \in (0, 1/2),$$

$$\hat{\gamma}_i(h) = T^{-1}\sum_{i=1}^{T-|h|}\left(\hat{e}_{i,t}^2 - \hat{v}_i\right)\left(\hat{e}_{i,t+|h|}^2 - \hat{v}_i\right), \quad \text{for } |h| < T,$$

where

$$\hat{e}_{i,t} = X_{i,t} - \bar{X}_T(i), \quad \bar{X}_T(i) = \frac{1}{T}\sum_{i=1}^{T}X_{i,t},$$

$$\hat{v}_i = T^{-1}\sum_{t=1}^{T}\hat{e}_{i,t}^2.$$

Lemma 1. *If (3) and (6) hold, and assume that a sequence of positive integers $\{h_T\}$ satisfies $h_T \to \infty$, $h_T = O\left(T^{\theta}\right)$ for some $\theta \in (0, 1/2)$. Then, as $T \to \infty$,*

$$\bar{\varphi}_i^2 = \sum_{|h|\le h_T}\bar{\gamma}_i(h) \xrightarrow{P} \varphi_i^2, \quad 1 \le i \le N,$$

$$\sup_{0\le x\le 1}\frac{1}{\bar{\varphi}_i}\frac{1}{\sqrt{T}}\left|\sum_{t=1}^{\lfloor Tx\rfloor}e_{i,t}^2 - \frac{\lfloor Tx\rfloor}{T}\sum_{t=1}^{T}e_{i,t}^2\right| \xrightarrow{d} \sup_{0\le x\le 1}\left|B^0(x)\right|, \quad 1 \le i \le N,$$

where $\bar{\gamma}_i(h) = T^{-1}\sum_{i=1}^{T-|h|}\left(e_{i,t}^2 - \bar{v}_i\right)\left(e_{i,t+|h|}^2 - \bar{v}_i\right)$, $\bar{v}_i = T^{-1}\sum_{t=1}^{T} e_{i,t}^2$, B^0 denotes a standard Brownian bridge. The existence of φ_i^2 is guaranteed by Lee and Park (2001).

Theorem 1. *If H_0, (3)–(7) hold, and*

$$\frac{N}{T} \to 0 \tag{11}$$

as $\min(N, T) \to \infty$, then

$$\sup_{0\le x\le 1}\left|V_{N,T}(x)\right| \xrightarrow{\mathcal{D}[0,1]} \sup_{0\le x\le 1}\left|B^0(x)\right|, \quad 0 \le x \le 1,$$

where $B^0(x)$ is a standard Brownian bridge, $\xrightarrow{\mathcal{D}[0,1]}$ denote the weak convergence of stochastic process in the Skorokhod space $\mathcal{D}[0, 1]$.

Theorem 2. *If H_1, (3)–(7) hold, and*

$$\frac{T^{1/2}}{N^{1/2}}\sum_{i=1}^{N}\delta_i \to \infty, \quad 1 \le i \le N,$$

as $\min(N, T) \to \infty$, then

$$\sup_{0\le x\le 1}\left|V_{N,T}(x)\right| \xrightarrow{P} \infty.$$

4. Monte Carlo simulations

In this section, we evaluate the finite sample performance of $\sup_{0\le x\le 1}\left|V_{N,T}(x)\right|$ through a simulation study, where $V_{N,T}(x)$ is defined in (10). The critical values of $\sup_{0\le x\le 1}\left|B^0(x)\right|$ for $\alpha = 0.1$, 0.05, 0.01 are 1.224, 1.358 and 1.628, respectively (cf. Inclan and Tiao, 1994). The data generating process are based on independent identically distributed random variables (standard normal and t distribution with 5 degrees of freedoms) and dependent random variables (AR(1) process). The series $N = 50, 100, 200$, the number of observations $T = 50, 100, 200$, and $h_T = T^{1/3}$ are used for $\hat{\varphi}_i^2$. All simulations are based on 2000 replications.

Tables 1 and 2 report the empirical sizes of $\sup_{0\le x\le 1}\left|V_{N,T}(x)\right|$ under the null hypothesis. Table 3 shows that the empirical powers under the alternative hypothesis: the innovations change from $N(0, 1)$ to t_5 at time t_0, where $t_0 = T/3, T/2$. The empirical sizes and powers are both calculated at the nominal level $\alpha = 0.05$. From Tables 1 and 2, it can be seen that there exist certain size distortions when the sample size N and T are small, the empirical sizes approach to the nominal level as N and T increase. The empirical powers of CUSUM test statistic are close to 1 even in case of small sample size, which can be seen in Tables 3.

5. Conclusion

In this paper, we propose a CUSUM based test for the variance of the panel data models. The asymptotic properties of our test under both the null of no change and the alternative of change in variance are proved. Monte Carlo simulation results demonstrate the proposed method is efficient.