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Why are losses from trade unlikely?

Igor Bykadorov [a,b], Alexey Gorn [c], Sergey Kokovin [a,b,*], Evgeny Zhelobodko [b,a,1]

[a] National Research University Higher School of Economics, Russia
[b] Novosibirsk State University, Russia
[c] Bocconi University, Italy

HIGHLIGHTS

- The general New Trade model with variable costs/substitution is explored.
- The necessary and sufficient condition for trade losses is found.
- The condition means "misaligned" preferences under specific costs.
- Numerical examples show that this case is possible but unlikely.

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ABSTRACT

Examining a standard monopolistic competition model with unspecified utility/cost functions, we find *necessary and sufficient* conditions on their elasticities for welfare losses to arise from trade or market expansion. Two numerical examples explain the losses (under unrealistic elasticities).

Introduction

Gains from trade and large markets are an important issue in monopolistic competition theory (Melitz and Redding, 2012), whereas possible *losses* are less studied, unlike in oligopoly settings (Brander and Krugman, 1983). Trying to prove the impossibility of harmful trade, we arrive instead at two counter-examples and a *criterion* (necessary and sufficient condition) for losses. The objective is to distinguish industries likely or unlikely to be harmed by globalization, by examining properties of their demand and supply functions.

This goal requires advanced modeling: variable elasticity of substitution (VES), unspecified preferences and general-form costs. Our setting deviates from Zhelobodko et al. (2012) by allowing both convex and *concave* total cost. This generalization is needed for an important feature: indirect modeling of endogenous technology (R&D). Indeed, when R&D is possible, higher output fosters investment in marginal cost *reduction*, which implies concave cost (Bykadorov et al., 2013).

The main result is condition (9) on utilities/costs, necessary and sufficient for intra-sectoral trade gains or losses in a generalized Dixit–Stiglitz–Krugman model. In addition, two numerical examples demonstrate that this requirement is plausible, i.e., compatible with other reasonable properties of preferences and costs. Therefore both directions of market distortion appear as theoretically possible: *excessive or insufficient entry can be aggravated by market growth*. However, utilities/costs that satisfy (9) are uncommon, and the related discussion shows why trade losses are *unlikely* in the real economy.

* Correspondence to: National Research University Higher School of Economics, ul. Pechatnikov, 16, St. Petersburg 190068, Russia. Tel.: +7 8 812 6779384; fax: +7 8 812 6779384.
E-mail addresses: bykadorov.igor@mail.ru (I. Bykadorov), alexey.gorn@gmail.com (A. Gorn), skokov7@gmail.com (S. Kokovin).

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1. Model

The model exposition follows Zhelobodko et al. (2012) to ease comparison. Our closed economy exhibits monopolistic competition under *unspecified* additive utility and cost functions, with variable marginal costs/elasticities. The only production factor is labor, supplied inelastically by L identical consumers/workers. A single sector involves an endogenous interval $[0, N]$ of identical firms producing varieties, one variety per firm.

Each consumer maximizes utility in the form

$$U = \int_0^N u(x_i)di \to \max_{X\geq 0}, \quad \text{s.t.} \int_0^N p_i x_i di \leq 1. \tag{1}$$

Here $X = (x_j)_{j\leq N}$ is a function, x_i denotes consumer's consumption of ith variety, p_i is the price, $w \equiv 1$ is wage, index $_i$ everywhere replaces parentheses (i). As in Zhelobodko et al. (2012), we use the elasticity operator $\mathcal{E}_g(z) \equiv \frac{zg'(z)}{g(z)}$ defined for any function g, and the Arrow–Pratt concavity operator $r_g(z) \equiv -\frac{zg''(z)}{g'(z)} = -\mathcal{E}_{g'}(z)$.

For existence, uniqueness and symmetry of the equilibrium, we make the following weak restrictions on utility (Zhelobodko et al., 2012; Mrázová and Neary, 2014). At some zone $[0, \check{z})$ of possible equilibria ($\check{z} \leq \infty$), the elementary utility function $u(\cdot)$ is thrice differentiable – increasing ($u'(z) > 0$), strictly concave ($u''(z) < 0$), normalized ($u(0) = 0$) – and its main characteristics behave as $r_u(z) \in [0, 1), \ r_{u'}(z) < 2\, \forall z \in [0, \check{z})$.

Then the first-order condition (FOC) with a Lagrange multiplier λ entails the inverse demand function $\mathbf{p}$ for any variety i:

$$\mathbf{p}(x_i, \lambda) = \frac{u'(x_i)}{\lambda}. \tag{2}$$

The marginal utility of income λ serves as the single market aggregate.

Each producer faces some total cost function $C(q)$ depending upon output $q \equiv Lx$, perceives function $\mathbf{p}$ and λ as given, and maximizes profit

$$\pi\ (x, \lambda) \equiv \frac{u'\ (x)}{\lambda} xL - C\ (Lx) \to \max_{x\geq 0}.$$

(Here, choice of maximizers x, q or p brings an equivalent result, and the firm's index i is dropped by symmetry.) Denoting revenue $R\ (x, \lambda, L) \equiv u'\ (x)\, xL/\lambda$, we can formulate the FOC in usual terms of marginal revenue and marginal cost: $\frac{d}{dx} R\ (x, \lambda, L) - \frac{d}{dx} C\ (Lx) = 0$. The second-order condition (SOC) is

$$-\frac{d^2\pi}{d^2x} = -\frac{d^2}{d^2x} R\ (x, \lambda, L) + \frac{d^2}{d^2x} C\ (Lx) > 0.$$

This condition yields symmetry of the equilibrium.

Equilibrium is a bundle $(\bar{x}, \bar{p}, \bar{\lambda}, \bar{N})$ satisfying the utility maximization condition (2); profit maximization FOC and SOC; free-entry and labor market clearing conditions:

$$R\left(\bar{x}, \bar{\lambda}, L\right) - C\ (L\bar{x}) = 0, \tag{3}$$

$$\bar{N} C\ (L\bar{x}) = L. \tag{4}$$

(Upper bar henceforth denotes equilibria.)

Now we can divide each producer's FOC by the free-entry condition to express our equilibrium through the elasticity of revenue $\mathcal{E}_R$, the elasticity of inverse demand $\mathcal{E}_{\mathbf{p}}(x) \equiv \frac{x}{\mathbf{p}} \cdot \frac{\partial \mathbf{p}(x)}{\partial x} \equiv -r_u(x)$ and the cost elasticity $\mathcal{E}_C(q) \equiv \frac{q}{C} \cdot \frac{\partial C(q)}{\partial q}$:

$$\mathcal{E}_R(\bar{x}) \equiv 1 - r_u(\bar{x}) = \mathcal{E}_C(L\bar{x}). \tag{5}$$

The equilibrium consumption $\bar{x}$ is determined here, whereas equilibrium prices $\bar{p}$ and mass $\bar{N}$ of firms can be found from the remaining equations. Therefore, each consumer's equilibrium welfare $\bar{U} = \bar{N}u(\bar{x})$ depends indirectly on market size L through the equilibrium magnitudes $\bar{x}(L), \bar{N}(L)$.

Totally differentiating the equilibrium equation (5) w.r.t. population size L and using (4), we express total utility elasticity $E_{\bar{U}/L}$ at equilibrium through other total elasticities $E_{\bar{N}/L} \equiv \frac{L}{\bar{N}} \cdot \frac{d\bar{N}}{dL}$, $E_{\bar{x}/L} \equiv \frac{L}{\bar{x}} \cdot \frac{d\bar{x}}{dL}$ and partial elasticity $\mathcal{E}_u \equiv \mathcal{E}_u(\bar{x}) \equiv \frac{z}{u(z)} \cdot \frac{\partial u(z)}{\partial z}$ as follows:

$$E_{\bar{U}/L} \equiv \frac{L}{\bar{U}} \cdot \frac{d\bar{U}}{dL} = E_{\bar{N}/L} + \mathcal{E}_u \cdot E_{\bar{x}/L}. \tag{6}$$

The SOC for profit maximization at equilibrium is

$$SOC \equiv r_u'(\bar{x}) \cdot \bar{x} + \mathcal{E}_C'(L\bar{x}) \cdot L\bar{x} > 0. \tag{7}$$

(Proofs are in Bykadorov et al. (2014).)

2. Losses from market size

Lemma. *The local effect of a growing market on welfare can be expressed in elasticities (taken at the equilibrium values) as follows:*

$$E_{\bar{U}/L} = (1 - \mathcal{E}_u) - \frac{\bar{x}^2}{\mathcal{E}_u} \cdot \frac{\mathcal{E}_u' \cdot r_u'}{SOC} = r_u + \frac{L\bar{x}^2}{\mathcal{E}_u} \cdot \frac{\mathcal{E}_u' \cdot \mathcal{E}_C'}{SOC}. \tag{8}$$

This lemma enables us to establish the necessary and sufficient condition for "harmful trade" through the following claims, each highlighting some aspect of market distortion.

Proposition. *Consider an equilibrium $\bar{x}$ under market size L_0. Any local welfare reduction caused by a growing market is equivalent to the following conditions on utility, revenue and cost elasticities:*

$$E_{\bar{U}/L} < 0 \Leftrightarrow \mathcal{E}_R'(\bar{x}) < \mathcal{E}_C'(L_0\bar{x}) \cdot L_0 < \mathcal{E}_R'(\bar{x}) \cdot \frac{r_u(\bar{x})}{1 - \mathcal{E}_u(\bar{x})}. \tag{9}$$

For a more convenient interpretation, this double inequality can be reformulated as follows.

Corollary. (i) [Necessity]. *For any welfare reduction two conditions are necessary:*

$$\mathcal{E}_u'(\bar{x}) \cdot \mathcal{E}_R'(\bar{x}) < 0, \tag{10}$$

$$\mathcal{E}_u'(\bar{x}) \cdot \mathcal{E}_C'(L\bar{x}) < 0. \tag{11}$$

In particular, under convex cost ($\mathcal{E}_C' > 0$), such reduction requires both increasingly elastic revenue (IER) and decreasingly elastic utility (DEU).

(ii) [Sufficiency]. *For any utility satisfying inequality (10) at some $\bar{x}$ under given L_0, one can find a cost function C such that $\bar{x}$ is an equilibrium, and welfare locally decreases w.r.t. L at L_0. One can find also another cost function $\tilde{C}$ that makes welfare locally increasing.*

Discussion. Under properties (9)–(11) holding globally, these claims are easily extended from infinitesimal changes in population and welfare ($\frac{d\bar{U}}{dL}$) onto *global* ones ($\frac{\Delta\bar{U}}{\Delta L}$).

Why are equilibria satisfying all conditions (9)–(11) *unlikely*? Property $\mathcal{E}_R'(x) \equiv -r_u'(x) < 0$ is called decreasingly elastic revenue (DER), being equivalent to increasingly elastic (strictly subconvex) demand (Mrázová and Neary, 2013). The DER case is called realistic by Krugman (1979) and subsequent papers (see Zhelobodko et al., 2012) because it generates decreasing prices under increasing competition; DER is perceived as "Marshall's Second Law of Demand" by Mrázová and Neary (2013). Then, (9) becomes $1 > \frac{-\mathcal{E}_C'(L\bar{x}) \cdot L}{r_u'(\bar{x})} > \frac{r_u(\bar{x})}{1-\mathcal{E}_u(\bar{x})}$. To get losses, some $C(.)$ must fit this double inequality, compatible only when $\mathcal{E}_u < 1 - r_u = \mathcal{E}_R$, which is