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Introducing consumer heterogeneity in dynamic games with multi-product firms and differentiated product demand

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HIGHLIGHTS

- We study dynamic games with multi-product firms and differentiated product demand.
- We propose conditions allowing for consumer heterogeneity.
- These conditions lead to state space reduction to a single state variable for each firm.

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ABSTRACT

This paper extends a discrete-choice model of differentiated product demand to consider consumer heterogeneity in dynamic games. Our approach applies to games involving both multi-product firms and static price competition.

1. Introduction

The study of oligopoly markets where some competition dimensions are dynamic has improved considerably since the seminal work of Ericson and Pakes (1995, henceforth EP). Most advances include estimation of dynamic game parameters, while equilibrium computations are being avoided (e.g., Aguirregabiria and Mira, 2007, Bajari et al., 2007, Pesendorfer and Schmidt-Dengler, 2008 and Pakes et al., 2007). However, many applications of dynamic models, such as counterfactual simulation, require equilibrium calculations. A "curse of dimensionality" renders these calculations computationally burdensome, unless the state space is small (Pakes and McGuire, 1994). Despite recent advances in computing EP-like models, alleviating the computational burden in calculating dynamic equilibria by state space reduction remains a challenge (Doraszelski and Pakes, 2007). A related important challenge is consumer heterogeneity in dynamic games (Aguirregabiria and Nevo, 2012).

Considering consumer heterogeneity in modeling differentiated product demand is crucial to obtain realistic predictions of key economic quantities, such as price elasticities and firm profits (e.g. Berry et al., 1995, Nevo, 2001 and Ackerberg et al., 2007). However, most research on dynamic competition assumes common demand parameters among consumers (e.g. Benkard, 2004 and Aguirregabiria and Ho, 2012).[1] If consumer tastes follow a certain distribution, price optimality conditions usually result in product-specific equilibrium prices. In this situation, a dynamic model of competition requires both product-specific state variables and equilibrium

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[1] One exception is the model by Sweeting (2013) regarding dynamic product repositioning without price competition.

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price computations, becoming intractable when applied to realistic cases such as markets with many products or comprising multi-product firms (Nevo and Rossi, 2008). In this paper, we propose a tractable extension of the EP framework that considers consumer heterogeneity. Our expansion of the Nested Logit model flexibly accommodates consumer heterogeneity in preferences across nests, alleviating the computational burden by means of both a single state variable for each firm and a reduced equilibrium price system in markets with many products or comprising multi-product firms.

2. The dynamic model

We consider a differentiated product version of the EP model similar to that of Pakes and McGuire (1994) and Nevo and Rossi (2008). While the model can allow for firm entry and exit, we focus on dynamic investment decisions.

2.1. Flow payoffs

In each period $t = 0, 1, \ldots, \infty$, M consumers choose at most one product to maximize utility. The utility consumer i derived from product j at period t is

$$U_{ijt} = \omega_{jt} - p_{jt} + \varepsilon_{ijt} \tag{1}$$

where $\omega_{jt} \in \Omega$ is the quality (or efficiency level) of product j, p_{jt} denotes its price, and ε_{ijt} is an idiosyncratic preference shock.[2] Ω is a discrete, finite set of quality states. The utility of no-purchase ($j = 0$) is normalized to $U_{i0t} = \varepsilon_{i0t}$.

F firms operate in the market. Each firm owns a subset of the available J products, which is denoted as $\mathcal{F}_f$. The market structure of the industry is characterized by the state vector $s_t = (\omega_{1t}, \ldots, \omega_{Jt})$. With the product qualities of a firm, s_t defines market prices and quantities. The flow profit of firm f is

$$\pi_f\left(\left\{\omega_{jt}\right\}_{j\in\mathcal{F}_f}, s_t\right) = \mathrm{M}\sum_{j\in\mathcal{F}_f}\left[p_j\left(\left\{\omega_{jt}\right\}_{j\in\mathcal{F}_f}, s_t\right) - mc_{jt}\right] \times \sigma_j\left(\left\{\omega_{jt}\right\}_{j\in\mathcal{F}_f}, s_t\right) - C_f \tag{2}$$

where p_j and σ_j represent product price and market share, respectively, C_f represents fixed costs, and mc_{jt} denotes product marginal cost. We assume that firm pricing decisions are static, aim at profit maximization, and result in a pure-strategy Nash–Bertrand equilibrium. This assumption is key for our results to hold, and is made by much of the literature we cite (e.g. Berry et al., 1995 and Nevo, 2001). Flow profits (2) include no dynamic components of period profits, which we address in the next section.

2.2. Dynamic controls and state transitions

In addition to prices, each firm chooses product investments x_{jt}, $j \in \mathcal{F}_f$ that are aimed at increasing the quality of each product in period $t + 1$. The outcome of the product investment of the firm is governed by the Markovian process:

$$\omega_{j,t+1} = \omega_{j,t} + \upsilon_{j,t} - \zeta_t. \tag{3}$$

The non-negative component $\upsilon_{j,t}$ represents the stochastic dependence of $\omega_{j,t+1}$ on x_{jt}. It follows a distribution $P_\upsilon(.|x)$, $x \in \mathbb{R}^+$, with properties of (i) $\upsilon_{j,t} = 0$ if $x_{j,t} = 0$ and (ii) stochastic increase in $x_{j,t}$. The time-specific demand shock ζ_t is a non-negative random variable assumed to be independent from $\upsilon_{j,t}$ and exogenous with probability measure $\mu(\zeta)$. These conditions are posed for expositional convenience and can be relaxed.

Firms choose investments to maximize their discounted flow of payoffs

$$V_f\left(\left\{\omega_{jt}\right\}_{j\in\mathcal{F}_f}, s_t\right) = \max_{\{x_{jt}\geq 0,\, j\in\mathcal{F}_f\}} \sum_{\tau=0}^{\infty}\delta^\tau \mathrm{E}\left\{\pi_f\left(\left\{\omega_{j,t+\tau}\right\}_{j\in\mathcal{F}_f}, s_{t+\tau}\right) - \mathrm{c}\sum_{j\in\mathcal{F}_f} x_{j,t+\tau}\right\} \tag{4}$$

where δ is a discount factor and c denotes the investment unit cost.[3] The beliefs of firm f on the state values of the succeeding period are summarized by the conditional distribution

$$P\left(\left\{\omega_{j,t+1}\right\}_{j\in\mathcal{F}_f}, s_{t+1}\middle|\left\{\omega_{jt}\right\}_{j\in\mathcal{F}_f}, s_t;\left\{x_{jt}\right\}_{j\in\mathcal{F}_f}\right) \tag{5}$$

that is used to compute the expected value $E\{\cdot\}$.

We assume a Markov Perfect Equilibrium whereby firms' investment strategies depend only on payoff-relevant state variables. Eqs. (4) and (5) are functions of all firms' strategies since the stochastic process governing the vector of firms' qualities depends on strategy functions.

The model is formally a complete information game with continuous controls and a finite state space. Provided that (5) is continuous in firms' strategies, standard arguments (e.g. Whitt, 1980) guarantee equilibrium existence in mixed strategies. Sufficient conditions for pure-strategy equilibrium existence can be provided in model extensions with private information shocks and entry/exit decisions (Doraszelski and Satterthwaite, 2010).

Solving for (4) at every combination of the state of the firm and that of the industry is computationally challenging because of the "curse of dimensionality". Nevo and Rossi (2008) simplify the problem by assuming that ε_{ijt} in (1) is identically and independently distributed type I extreme value. Under the Nash–Bertrand equilibrium condition, this assumption implies an optimal pricing rule where each firm applies the same markup to all of its products. Using this result, Nevo and Rossi (2008) show that equilibrium markups, market shares and firm flow profits depend only on what they name as firm-specific, adjusted inclusive values (AIVs). The AIV shares similarities with the inclusive value (IV) of McFadden (1978), which measures the expected utility from a group of products prior to observing the preference shocks ε_{ijt}'s. Formally, the AIV is equivalent to an IV, defined below, except that (i) product prices are replaced with marginal costs of production and (ii) $\mathcal{F}_f$ is the relevant group of products. Under an additional assumption relating state transitions and total firm-level investment, described below, the state space dimension of the firm problem in (4) reduces from J product-specific state variables to F AIVs.

The AIV approach has the advantage of providing a consistent framework where state variables are not ad hoc but are directly derived from the model. Importantly, other candidate state variables such as market shares or variable profits usually require stronger assumptions. For example, market shares alone cannot account for differences in product marginal costs necessary to compute profits, while the AIV approach deals with this issue in the way described above.[4]

[2] ω_{jt} is frequently expressed as $\omega_{jt} = X_{jt}\beta + \xi_{jt}$ where X_{jt} represents observable product characteristics, ξ_{jt} denotes attributes unobserved by the researcher, and β is a parameter vector.

[3] Assuming constant investment cost simplifies our exposition, yet more general cost functions could be considered.

[4] For an example of additional structure on variable profit transitions, see Aguirregabiria and Ho (2012).