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Participation and exclusion in auctions

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HIGHLIGHTS

- We compare auctioneer payoffs and revenues due to buyer participation and exclusion.
- An example shows the failure of the Bulow–Klemperer Theorem due to seller valuation.
- Extension of the BK Theorem with a minimum reserve price.
- Extension of the BK Theorem with a minimum buyer participation.
- A new tool is developed to establish new results and for further applications.

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ABSTRACT

We provide extensions of the Bulow and Klemperer (1996) result when the seller has value for the object above the minimum value of the buyers. The result may fail. We show that the seller does better with more participation and some exclusion than the optimal exclusion of buyers of low value types. Some amount of exclusion, which is independent of the number of buyers, in the form of the minimum bid is needed to make participation the dominant method for improving the seller payoff from the auctions. There exists N_0, which depends on the seller valuation, such that more participation with no exclusion is dominant if and only if the number of participants exceeds N_0.

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1. Introduction

The seller can raise the revenue in an auction in two ways. One is to increase participation of more buyers. The other is to exclude the low types of the existing buyers from winning. When the seller has no value for the object, we can interpret the result of Bulow and Klemperer (1996) as saying that the seller can do better with the participation of one more buyer without any exclusion of low-value buyers, and the payoff is higher than figuring out how to optimally exclude the low-types among existing buyers.

This result is not true in general when the seller has value for the object herself. We give a simple example to show that more participation need not do better than optimal exclusion.[1] In practice, it is common for a seller to set a reserve price in an auction to exclude the low-value buyers from winning. We show that with some amount of exclusion, more participation is better than the optimal exclusion. The amount of exclusion is fixed, independent of the number of the participants. There is strong empirical evidence that the reserve price is set lower than the optimal one. For an overview of this literature, see Ostrovsky and Schwarz (2009). This can be explained by the seller's desire to focus on more participation rather than the optimal exclusion which often requires detailed knowledge of the distribution of the buyer valuations.

We show that there exists a number of buyers N_0, such that more participation without any exclusion is better than the optimal exclusion if and only if $N \geq N_0$. The number N_0 depends on the seller valuation v_s. When v_s is below the minimum buyer valuation α, we can take $N_0 = 1$. When $v_s > \alpha$, for the Bulow–Klemperer result to hold, either some amount of exclusion is needed, or some amount of participation is needed without any exclusion.

A good intuition of the original Bulow–Klemperer result is in Kirkegaard (2006). He argued that the standard ascending auction

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1 Another situation this may occur is when there is renegotiation in Wang (2000). See Bulow and Klemperer (2009) for a related issue when entry is costly.

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with no reserve price is constrained optimal (subject to the sale of the object with probability one). If we perform an optimal auction, followed by giving it free to the new buyer when the object is not sold, then we get the same revenue as the optimal auction and this revenue is dominated by the constrained optimal one. Kirkegaard's argument does not apply when the seller has value herself. There is no clear way of formulating a constrained optimal auction which applies to the ascending auction with a reserve price equal to the seller's own value. For example, the ascending auction with a positive reserve price is not constrained optimal subject to the price being at least equal to the seller value. It is not constrained optimal either subject to the probability of sale being at least equal to that of the ascending auction. This will be shown later by an example.

The original proof in Bulow and Klemperer (1996) and Milgrom (2004, Chapter 4) is based on Jensen's inequality. It requires that the expected virtual value should be at least zero (or equal to the minimum buyer value). This requirement does not hold when applied to the payoff contribution. The expected payoff contribution (virtual value minus seller value) of a buyer is negative.

Our proof for the extension of the Bulow–Klemperer result relies on a single-crossing property of the payoff difference as a function of the number of buyers. This analytical tool is rather powerful, and has been successfully used in extending the Bulow–Klemperer result to more complex environments such as the case when the new buyer is recruited from the secondary market (see Cheng, 2015). Bidder participation and exclusion effects have become useful in econometric testing of bidder entry models and estimation of revenue gains from auction design in Coey et al. (2014). Since in real world data, we expect the seller valuation to be a relevant factor, our work can have interesting implications in that direction.

Some interesting questions can be raised. What is the typical amount of exclusion needed? What is the typical amount of participation needed? To what extent can the low reserve price in practice be explained by the focus on participation rather than exclusion?

2. Statement of the results

We focus on the case of independent private values (IPV). Let v_s be the seller value for the object for sale. Assume that there are initially N buyers, each has the value distribution $F(v)$ defined and C^2 differentiable over $[\alpha, \beta]$ with $F(\alpha) = 0, f(x) > 0$ when $x \in (\alpha, \beta]$. Let $J(x) = x - \frac{1-F(x)}{f(x)}$ be the virtual value (or marginal revenue) of the buyer, and ρ^* be the optimal reserve price from Myerson (1981). The optimal reserve price is determined by the equation

$$J(\rho^*) = v_s. \tag{1}$$

when it is above α. Assume that $J(x)$ is an increasing function. We have

$$\int_\alpha^\beta J(x)dF(x) = 0. \tag{2}$$

In this symmetric framework, we have revenue equivalence from Myerson (1981), so we do not need to specify the auction format used, as the payoff of the seller is independent of the auction format.

When v_s is positive, or exceeds α, we need to consider the seller's payoff rather than the revenue. No exclusion means that the seller uses his value as the reserve price. Let

$$U(N, \rho^*) = \int_{\rho^*}^\beta (J(x) - v_s)dF^N(x) \tag{3}$$

be the seller payoff from the optimal mechanism. Let

$$U(N+1, \rho) = \int_\rho^\beta (J(x) - v_s)dF^{N+1}(x) \tag{4}$$

be the seller payoff from the auction with one more participant and reserve price ρ.

We give an example showing that the Bulow–Klemperer result may fail when $v_s > \alpha$. Let the buyer value be uniformly distributed between [0, 1]. Assume initially that there is only one buyer. Let the seller value be $v_s = 0.4$. The seller can set the optimal reserve price $\rho^* = 0.7$ with the optimal payoff

$$U(1, \rho^*) = (0.7 - 0.4)(1 - 0.7) = 0.09.$$

With the participation of one more buyer, and $\rho = 0.4$, the seller payoff is

$$U(2, v_s) = \int_{0.4}^1 (2x - 1.4)dx^2 = 0.072 < 0.09.$$

Hence the Bulow–Klemperer result does not hold in this example. Note that the expected value of the winner with two buyers is higher with more participation than with optimal exclusion. However, the expected contribution to the seller's payoff is lower.

The auction with the reserve price $\rho = v_s$ is not constrained optimal subject to the selling price to be at least v_s. This is because the optimal auction to one buyer followed by selling the object to the new buyer at the price v_s satisfies the constraint, but yields higher payoff to the seller. It is not constrained optimal subject to the probability of sale being at least $1 - 0.4^2 = 0.84$. The probability of sale for the optimal auction to one buyer is 0.3. The seller can offer the object to the new buyer at the price 0.46. The total probability of sale is $0.3 + 1 - 0.46 = 0.84$. Therefore this selling strategy gets the object sold at probability 0.84, and yields higher payoff to the seller than the ascending auction with the reserve price 0.4.

Define the number ρ_0 to be the smallest number satisfying the following inequality in $\rho \in [\alpha, \rho^*]$:

$$U(2, \rho) = \int_\rho^\beta (J(x) - v_s)dF^2(x) \geq U(1, \rho^*). \tag{5}$$

The left-hand side of (5) is increasing in ρ, and equal to the optimal payoff with two buyers at $\rho = \rho^*$, which is higher than the right-hand side. Hence the left-hand side is either lower at $\rho = \alpha$, thus yielding a unique solution ρ_0 to the equation

$$\int_\rho^\beta (J(x) - v_s)dF^2(x) = U(1, \rho^*). \tag{6}$$

Or it is higher at $\rho = \alpha$, and in this case $\rho_0 = \alpha$.

We have the following result, the first part of which is the case considered in Bulow and Klemperer (1996). The second part is new, and says that more participation with the minimum bid ρ_0 is better than the optimal exclusion.

Theorem 1. (i) *When $v_s \leq \alpha$, we have*

$$U(N+1, \alpha) > U(N, \rho^*),$$

for all $N \geq 0$. (ii) *When $v_s \in (\alpha, \beta)$, we have*

$$U(N+1, \rho_0) > U(N, \rho^*)$$

for all $N \geq 1$, where $\rho_0 < \rho^*$ is defined in (5).

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