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Estimation of spatial panel data models with time varying spatial weights matrices[☆]

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HIGHLIGHTS

- We investigate spatial panel data models with time varying spatial weights matrices.
- Asymptotic properties of QMLE is derived.
- Estimation and inference for the impact analysis are studied.
- Simulations show that parameter and impact estimators have satisfactory performance.
- Simulations show that misspecification of weights matrices causes substantial biases.

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ABSTRACT

This paper investigates the quasi-maximum likelihood (QML) estimation of spatial panel data models where spatial weights matrices can be time varying. We show that QML estimate is consistent and asymptotically normal. We also derive the asymptotic distribution of average impact coefficients (direct, indirect, total). Monte Carlo results are reported to investigate the finite sample properties of QML estimates and impact coefficients.

1. Introduction

For spatial panel data models, the spatial weights matrix can be constructed from economic/socioeconomic distances or demographic characteristics, which might be changing over time. For example, in panel data setting, Case et al. (1993) construct a weights matrix based on the difference in the percentage of the population that is black; Baicker (2005) constructs a weights matrix with the degree of population mobility between regions. One may wonder whether we can easily handle the models with time varying spatial weights, and whether ignoring time variation in spatial weights matrices would have substantial consequences on estimates. Lee and Yu (2012) investigate the time varying weights matrices in a dynamic spatial panel model setting, where the number of time periods T is assumed to be large. In the current paper, we consider the static spatial panel model with both individual and time fixed effects, where T could be finite or large.

For the estimation and statistical inference of impact effects, LeSage and Pace (2009) provide a computationally efficient simulation approach to produce empirical estimates of dispersion for scalar summary measures of impacts. Debarsy et al. (2012) extend the preceding approach to the dynamic spatial panel data models with a time invariant spatial weights matrix. Elhorst (2012) pro-

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vides Matlab routines for the bias-corrected estimates in Lee and Yu (2010) and relevant impact analysis. The current paper will provide analysis for those impacts based on quasi-maximum likelihood (QML) estimators.

The rest of the paper is organized as follows. Section 2 introduces the model and establishes asymptotic properties of QML estimators. Section 3 investigates the impact estimates and their asymptotic inference. Section 4 provides Monte Carlo results. Section 5 concludes the paper. Due to space limit, lemmas and proofs are collected in a supplement file available upon request (see Appendix A).

2. The model and asymptotic properties of the QML estimate

The model considered is

$$Y_{nt} = \lambda_0 W_{nt} Y_{nt} + X_{nt}\beta_0 + \mathbf{c}_{n0} + \alpha_{t0} l_n + V_{nt},\quad t = 1, 2, \ldots, T, \tag{1}$$

where $Y_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$ and $V_{nt} = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ are $n \times 1$ column vectors, and v_{it}'s are *i.i.d.* across i and t with zero mean and variance σ_0^2. The X_{nt} is an $n \times K$ matrix of individually and time varying nonstochastic regressors, $\mathbf{c}_{n0}$ is an $n \times 1$ column vector of individual effects, and α_{t0} is the tth element of the $T \times 1$ fixed time effect vector $\boldsymbol{\alpha}_{T0}$ with l_n being $n \times 1$ vector of ones. The spatial weights matrix W_{nt} is nonstochastic and it could be time varying.[1] We assume that W_{nt} is row-normalized.[2]

Similar to Lee and Yu (2010), we can use the eigenvector matrix of $J_n = I_n - \frac{1}{n} l_n l_n'$ to eliminate the time effects. However, we will directly estimate the individual effects.[3] By denoting $S_{nt}(\lambda) = I_n - \lambda W_{nt}$ for an arbitrary λ, $\tilde{X}_{nt} = X_{nt} - \frac{1}{T}\sum_{t=1}^T X_{nt}$, and $\theta = (\beta', \lambda, \sigma^2)'$, the concentrated log likelihood (with $\mathbf{c}_n$ and α_t concentrated out) is

$$\ln L_{n,T}(\theta) = -\frac{(n-1)T}{2}\ln 2\pi - \frac{(n-1)T}{2}\ln\sigma^2 - T\ln(1-\lambda) + \sum_{t=1}^T \ln|S_{nt}(\lambda)| - \frac{1}{2\sigma^2}\sum_{t=1}^T \tilde{V}_{nt}'(\theta) J_n \tilde{V}_{nt}(\theta), \tag{2}$$

where $\tilde{V}_{nt}(\theta) = \widetilde{S_{nt}(\lambda)Y_{nt}} - \tilde{X}_{nt}\beta$ with $\widetilde{S_{nt}(\lambda)Y_{nt}} = S_{nt}(\lambda)Y_{nt} - \frac{1}{T}\sum_{t=1}^T S_{nt}(\lambda)Y_{nt}$ and $J_n\tilde{V}_{nt}(\theta) = J_n[\widetilde{S_{nt}(\lambda)Y_{nt}} - \tilde{X}_{nt}\beta - \tilde{\alpha}_t l_n]$ because $J_n l_n = \mathbf{0}$.

For asymptotic analysis of the QML estimators, we assume the following regularity conditions.

Assumption 1. W_{nt}'s are row-normalized nonstochastic spatial weights matrices with zero diagonals.

Assumption 2. The disturbances $\{v_{it}\}$, $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots, T$, are *i.i.d.* across i and t with zero mean, variance σ_0^2 and $\mathrm{E}\,|v_{it}|^{4+\eta} < \infty$ for some $\eta > 0$.

Assumption 3. The elements of X_{nt}, $\mathbf{c}_{n0}$ and $\boldsymbol{\alpha}_{T0}$ are nonstochastic and bounded, uniformly in n and t. Also, $\lim_{n\to\infty}\frac{1}{nT}\sum_{t=1}^T \tilde{X}_{nt}' J_n \tilde{X}_{nt}$ exists and is nonsingular.

Assumption 4. $S_{nt}(\lambda)$ is invertible for all t and for all $\lambda \in \Lambda$, where the parameter space Λ is compact and λ_0 is in the interior of Λ.

Assumption 5. W_{nt}'s and $S_{nt}^{-1}(\lambda)$'s are uniformly bounded (uniformly in t for W_{nt}'s, and uniformly in $\lambda \in \Lambda$ and t for $S_{nt}^{-1}(\lambda)$'s) in both row and column sums in absolute value.

Assumption 6. n is large, where T can be finite or large.

In Lee and Yu (2010) with time invariant weights matrix, the direct approach (estimating the individual effects directly) will yield bias for the variance parameter. Denote $\theta_T = \theta_0 - (\mathbf{0}_{1\times(K+1)}, \frac{1}{T}\sigma_0^2)'$. The asymptotic analysis for the direct approaches is based on θ_T. For the time varying spatial weights matrices case in the current paper, because we transform the data to eliminate the time effects but directly estimate the individual effects, we expect that the bias for the variance parameter remains. Thus, we will similarly base our asymptotic analysis on θ_T, and make bias correction for the variance parameter.

Denoting $G_{nt} = W_{nt}S_{nt}^{-1}$, $\widetilde{G_{nt}X_{nt}} = G_{nt}X_{nt} - \frac{1}{T}\sum_{t=1}^T G_{nt}X_{nt}$ and $\tilde{G}_{nt} = G_{nt} - \frac{1}{T}\sum_{t=1}^T G_{nt}$. The information matrix $\Sigma_{\theta_T,nT} = -\mathrm{E}\left(\frac{1}{(n-1)T}\frac{\partial^2 \ln L_{n,T}(\theta_T)}{\partial\theta\partial\theta'}\right)$ is equal to

$$\Sigma_{\theta_T,nT} = \frac{1}{\sigma_T^2}\begin{pmatrix} \mathrm{E}\mathcal{H}_{nT}^c & * \\ \mathbf{0}_{1\times(K+1)} & 0 \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{K\times K} & * & * \\ \mathbf{0}_{1\times K} & \frac{1}{(n-1)T}\sum_{t=1}^T\left[\mathrm{tr}(G_{nt}'J_nG_{nt}) + \mathrm{tr}((J_nG_{nt})^2)\right] & * \\ \mathbf{0}_{1\times K} & \frac{1}{\sigma_T^2(n-1)T}\sum_{t=1}^T \mathrm{tr}(J_nG_{nt}) & \frac{1}{2\sigma_T^4} \end{pmatrix}, \tag{3}$$

where $\mathcal{H}_{nT}^c = \frac{1}{(n-1)T}\sum_{t=1}^T(\tilde{X}_{nt}, (\widetilde{G_{nt}X_{nt}}\beta_0 + \tilde{G}_{nt}\mathbf{c}_{n0}))' J_n(\tilde{X}_{nt}, (\widetilde{G_{nt}X_{nt}}\beta_0 + \tilde{G}_{nt}\mathbf{c}_{n0}))$. The limit of $\Sigma_{\theta_T,nT}$ is nonsingular if $\lim_{n\to\infty}\mathrm{E}\mathcal{H}_{nT}^c$ is nonsingular or

$$\lim_{n\to\infty}\left(\frac{1}{(n-1)T}\sum_{t=1}^T\left[\mathrm{tr}(G_{nt}'J_nG_{nt}) + \mathrm{tr}((J_nG_{nt})^2)\right] - 2\left[\frac{1}{T}\sum_{t=1}^T\frac{\mathrm{tr}(J_nG_{nt})}{n-1}\right]^2\right) \neq 0. \tag{4}$$

For asymptotic distribution, denote by $\Omega_{\theta_T,nT}$ the equation given in Box I.

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[1] The elements of W_{nt} can be constructed from $h(z_{it}, z_{jt})$ where h is a bounded function and z_{it} contains some economic/socioeconomic distances information. Thus, the evolution of W_{nt} would be determined by the evolution of z_{it}. Also, if z_{it} is uncorrelated with v_{it}, W_{nt} is exogenous. If z_{it} is correlated with v_{it}, W_{nt} is then endogenous and relevant specification and estimation are beyond the scope of the current paper.

[2] Row normalization is convenient for parameter interpretation of λ; however, it usually changes the information content of the initial (non-normalized) weights matrix since each row is divided differently from the other. For instance, if the weights are based on inverse distance, row normalization will convert absolute distance into relative ones, which makes a big difference from the economic viewpoint (see Baltagi et al., 2008). There is also matrix normalization in Kelejian and Prucha (2010) that keeps the same information content and does not have such drawbacks. From estimation point of view for the panel data setting such as (1), if time effects $\alpha_{t0}l_n$ is not present, the estimation procedure in the current paper can be applied to both row normalization and matrix normalization cases. When time effects $\alpha_{t0}l_n$ is present, row normalization can still keep the SAR form of the model after the data transformation to eliminate the time effects; however, matrix normalization has to estimate the time effects directly so that the incidental parameter problem will occur with the bias magnitude $O(\frac{1}{n})$.

[3] We can eliminate the individual effects by eigenvector matrix of $J_T = I_T - \frac{1}{T}l_Tl_T'$. But, due to the time varying feature of spatial weights matrices, the transformed equation is no longer an SAR process and the QML approach cannot be applied directly. To see this, denote $(F_{T,T-1}, l_T/\sqrt{T})$ as the orthonormal matrix of eigenvectors of J_T and $[Y_{n1}^*, \ldots, Y_{n,T-1}^*] = [Y_{n1}, \ldots, Y_{nT}]F_{T,T-1}$ as the transformed dependent variables. When W_{nt} is time invariant, $[W_nY_{n1}, \ldots, W_nY_{nT}]F_{T,T-1} = W_n[Y_{n1}, \ldots, Y_{nT}]F_{T,T-1} = W_n[Y_{n1}^*, \ldots, Y_{n,T-1}^*]$ is the spatial lag of $[Y_{n1}^*, \ldots, Y_{n,T-1}^*]$ so that we still have the SAR form. When W_{nt} is time varying, $[W_{n1}Y_{n1}, \ldots, W_{nT}Y_{nT}]F_{T,T-1}$ cannot be written as a spatial lag of $[Y_{n1}^*, \ldots, Y_{n,T-1}^*]$. Thus, we will adopt a direct approach where we eliminate the time effects but estimate the individual effects directly.