Economics Letters 124 (2014) 329–334

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Recursive preferences, learning and large deviations

Chetan Dave [a,*], Kwok Ping Tsang [b]

[a] New York University (Abu Dhabi), PO Box 903, New York, NY 10276, USA

[b] Department of Economics, Virginia Tech, Blacksburg, VA, 24061, USA

HIGHLIGHTS

- An asset pricing model with recursive preferences is specified.
- The model is estimated under the assumption of adaptive learning.
- Both of these sources of volatility account for fluctuations in liquid stock markets.
- However, only risk aversion matters for illiquid housing markets.

ARTICLE INFO

Article history:
Received 10 May 2014
Received in revised form
10 June 2014
Accepted 15 June 2014
Available online 24 June 2014

JEL classification:
D80
D83
D84

Keywords:
Recursive preferences
Adaptive learning
Large deviations
Fat tails
Asset prices

ABSTRACT

We estimate the relative contribution of recursive preferences versus adaptive learning in accounting for the tail thickness of price–dividends/rents ratios. We find that both of these sources of volatility account for volatility in liquid (stocks) but not illiquid (housing) assets.

1. Introduction

In analyzing sources of volatility in asset pricing models two mechanisms stand out: learning and the adoption of recursive preferences. The former attributes volatility in asset prices to processes via which an agent comes to know of underlying fundamentals. The latter requires the agent to care about *when* uncertainty is resolved, determined entirely by preferences for risk and intertemporal substitution. Benhabib and Dave (2014) suggest that, in a single asset version of Lucas (1978), adaptive learning via a constant gain stochastic gradient (CGSG) algorithm causes the stationary distribution of the price–dividends ratio (PDR) to exhibit fat tails despite dividends being modeled as a thin-tailed process. These fat tails are shown to vary as a function of the deep parameters of the model, and the large deviations of the PDR from its rational expectations equilibrium value are able to account for the volatility in stock indices. Here we investigate the empirical contribution of a recursive preference formulation, following Epstein and Zin (1989, 1991), to the learning and large deviations model for both the stock and housing markets.

Why this particular formulation? The estimates provided in Benhabib and Dave (2014) of the CRRA coefficient are high. High estimates of the CRRA coefficient are common in empirical consumption asset pricing and warrant further investigation. Further, Benhabib and Dave (2014) show that the higher the CRRA coefficient, the thicker the tails of the stationary distribution of the PDR; it would be useful to have low CRRA estimates and still account for the thick tails due to a learning algorithm. Given the elegant manner in which recursive preferences separate out the effects of risk aversion and intertemporal substitution, perhaps

* Corresponding author.
E-mail addresses: cdave@nyu.edu (C. Dave), byront@vt.edu (K.P. Tsang).

http://dx.doi.org/10.1016/j.econlet.2014.06.014
0165-1765/

their adoption is sufficient to "break out" a high CRRA coefficient estimate into its constituent components whilst still retaining a role for learning. Further, simulations can indicate whether the tails of the stationary distribution of the PDR thicken as the parameterized preferences for risk *and* the desire to smooth consumption intertemporally increase. Finally, by comparing estimates from S&P 500 data versus an illiquid housing market, one can further focus on the three forces that could affect the thickness of the tail of a PDR or price–rent ratio (PRR) series: risk aversion, intertemporal substitution and learning.

Our investigation does deliver the sought-after results. Allowing for recursive preferences with S&P 500 data suggests an estimate for the CRRA coefficient of around 2.5 and an estimate for the inverse elasticity of intertemporal substitution of around 1.2. Further, as these parameters increase, the tail index of the PDR that governs the thickness of its tails does fall. The higher the CRRA coefficient or the inverse elasticity of intertemporal substitution, the smaller is the number of moments associated with the tail of the stationary distribution of the PDR. Finally, the estimate of the learning gain does not change much relative to Benhabib and Dave (2014), indicating that learning continues to play a strong role in determining volatility in the S&P 500. With housing data, we find that the estimate of the gain parameter, which in part governs the effect of learning on the tail of the stationary distribution of the PRR, comes with a very large standard error. However, the relative risk aversion parameter is precisely estimated. This suggests that for the housing market a main driver for the thickness of tail of the stationary PRR is risk aversion, an expected result given the illiquid nature of housing as an asset.

In the next section we specify the model under recursive preferences, and then provide minimum distance estimates of its parameters in Section 3. We discuss simulated "comparative statics" results in Section 4 and conclude in Section 5 with a description of how the formulation of models as linear recursions with multiplicative noise can assist in characterizing data (and models) that exhibit fat tails and thus the possibility of rare disasters.

2. The model

Under recursive preferences the representative agent optimizes

$$\left[(1-\beta)C_t^{1-\gamma} + \beta U_{t+1}^{*1-\gamma}\right]^{\frac{1}{1-\gamma}}, \quad \beta \in (0,1) \tag{1}$$

subject to the usual constraints, where β is the discount factor. The inverse of the parameter $\gamma > 0$ is the intertemporal elasticity of substitution. Certainty equivalent future utility is given by

$$U_{t+1}^* = \left[E_t\left(U_{t+1}^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} \tag{2}$$

where the parameter $\alpha > 0$ is the relative risk aversion coefficient.[1] The stochastic discount factor is

$$m_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{V_{t+1}}{U_{t+1}^*}\right)^{\gamma-\alpha}. \tag{3}$$

Defining wealth as the present value of consumption

$$W_t = C_t + E_t\left(m_{t,t+1}W_{t+1}\right) \tag{4}$$

the return to wealth is

$$R_{t,t+1} = \frac{W_{t+1}}{W_t - C_t} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{V_{t+1}}{U_{t+1}^*}\right)^{\gamma-1}\right]^{-1}, \tag{5}$$

yielding the stochastic discount factor

$$m_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{\frac{-\gamma(1-\alpha)}{1-\gamma}} R_{t,t+1}^{\frac{\gamma-\alpha}{1-\gamma}}. \tag{6}$$

If stocks are the only wealth, and consumption is the same as dividends (D_t) in equilibrium, we have the single asset pricing equation

$$1 = \beta E_t\left[\left(\frac{D_{t+1}}{D_t}\right)^{\frac{-\gamma(1-\alpha)}{1-\gamma}}\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right)^{\frac{1-\alpha}{1-\gamma}}\right]. \tag{7}$$

Let $\kappa = \frac{1-\alpha}{1-\gamma}$, then in the steady state

$$\overline{P} = \frac{\beta^{\frac{1}{\kappa}}}{1-\beta^{\frac{1}{\kappa}}}\overline{D}. \tag{8}$$

The linear approximation is

$$\begin{aligned} 0 \approx\ & \frac{\gamma\kappa\beta(\overline{P}+\overline{D})^{\kappa}}{\overline{D}}d_t \\ & + \left[-\gamma\kappa\beta(\overline{P}+\overline{D})^{\kappa}\overline{D} + \beta\kappa\overline{D}\left(\frac{\overline{P}+\overline{D}}{\overline{P}}\right)^{\kappa-1}\right]E_t d_{t+1} \\ & - \kappa\overline{P}^{\kappa}p_t + \kappa\beta(\overline{P}+\overline{D})^{\kappa-1}\overline{P}E_t p_{t+1}. \end{aligned} \tag{9}$$

We let $\overline{D} = 1$ to obtain the linearized equation

$$\begin{aligned} p_t &= \gamma d_t + \eta E_t(d_{t+1}) + \beta^{\frac{1}{\kappa}}E_t(p_{t+1}), \\ \eta &= \left[\left(1-\beta^{\frac{1}{\kappa}}\right)^{\kappa}\left(\beta^{\frac{1}{\kappa}}\right)^{1-\kappa}\right] \end{aligned} \tag{10}$$

where all lowercase variables denote log-deviations from steady state $(\overline{P},\overline{D}) = \left(\frac{\beta^{\frac{1}{\kappa}}}{1-\beta^{\frac{1}{\kappa}}}, 1\right)$.

Assume that the exogenous dividends process follows

$$d_t = \rho d_{t-1} + \varepsilon_t, \quad |\rho| < 1,\ \varepsilon_t \sim i.i.d.(0,\sigma_\varepsilon^2),\ \sigma_\varepsilon^2 < +\infty \tag{11}$$

with compact support $[-a, a]$, $a > 0$. Then from this process for dividends we know that

$$E_t(d_{t+1}) = \rho d_t \tag{12}$$

and so

$$\begin{aligned} p_t &= \beta^{\frac{1}{\kappa}}E_t(p_{t+1}) + \theta d_t, \quad \theta \equiv \gamma + \eta\rho, \\ \eta &= \left[\left(1-\beta^{\frac{1}{\kappa}}\right)^{\kappa}\left(\beta^{\frac{1}{\kappa}}\right)^{1-\kappa}\right], \quad \kappa = \frac{1-\alpha}{1-\gamma} \end{aligned} \tag{13}$$

is the fundamental expectational difference equation under investigation.

The REE value of ϕ is given by

$$\phi^{\text{REE}} = \frac{\theta\rho}{1-\beta^{\frac{1}{\kappa}}\rho} \tag{14}$$

which is unique and finite for all $\beta^{\frac{1}{\kappa}}\rho \neq 1$ so we assume that condition holds.

For learning we conjecture that

$$p_t = \phi_{t-1}d_{t-1} + \xi_t, \quad \xi_t \sim i.i.d.(0,\sigma_\xi^2),\ \sigma_\xi^2 < +\infty \tag{15}$$

implying

$$E_t(p_{t+1}) = \phi_{t-1}d_t \tag{16}$$

[1] When $\gamma = \alpha$ we have the standard time-separable CRRA preference specification.