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On the validity of the first-order approach with moral hazard and hidden assets

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HIGHLIGHTS

- If effort and assets are hidden, first-order conditions may not be sufficient.
- Previous research has derived appealing restrictions that imply sufficiency.
- These restrictions are valid for economies with a *single* hidden asset.
- Appealing restrictions fail to achieve sufficiency when *multiple* assets are hidden.

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ABSTRACT

With moral hazard and anonymous asset trade, first-order conditions need not characterize effort and portfolio choices. The standard procedure for establishing validity of the first-order approach in economies with *one* hidden asset is not fruitful when *multiple* assets are hidden.

1. Introduction

Insurance provision is constrained by asymmetric information. Obviously, more insurance decreases incentives to reduce the probability of bad outcomes by exerting unobservable effort. This moral hazard problem becomes more severe when agents not only privately choose effort but also hiddenly own assets that, in anonymous competitive equilibrium, trade at prices that neglect the effect of insurance on effort incentives.

Pauly (1974) uses first-order conditions to characterize this source of inefficiency when insurance is traded against a single event, so that only one asset position is private information. In that setting, Bertola and Koeniger (2013) derive functional form restrictions ensuring validity of that "first-order approach". Ábrahám et al. (2011) similarly establish that first-order conditions are necessary and sufficient, under sensible and interpretable functional restrictions, in an economy with exclusive formal insurance and a single hidden non-contingent asset.

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In this note we show that the method of these papers is not fruitful if agents can trade multiple hidden assets. This finding is not only a technicality since those and other recent papers build on the classic approach of Rogerson (1985) to establish validity of the first-order approach in moral hazard models. The case of multiple hidden assets is a natural extension of this existing literature, which is motivated by a general concern about asset observability that is not restricted to a single asset.

2. The problem

We assume that privately chosen effort e determines a non-degenerate probability distribution $f(z_i|e)$ for observable income realizations z_i, $i = 1, \ldots, n$, with $0 < f(z_i|e) < 1$.[1] It will be useful below to index z_i in increasing order, $z_1 < z_2 < \cdots < z_{n-1} < z_n$. Ex-ante identical individuals derive disutility $-v(e)$ from exerting effort and enjoy expected utility $\mathbf{E}u(c) = \sum_{i=1}^{n} u(c(z_i))f(z_i|e)$ from consumption of $c(z_i)$ upon realization of income z_i. To focus on the interior optima that may be characterized by first-order conditions, we assume that $v'(0) = 0$ and that consumption c can vary freely along the budget constraint.[2]

Atomistic individuals maximize the objective function

$$V\left(e, \{q(z_i)\}_{i=1}^{n}\right) = -v(e) + \sum_{i=1}^{n} u(c(z_i))f(z_i|e) \qquad (1)$$

$$\text{for } c(z_i) = z_i + q(z_i) - \sum_{j=1}^{n} q(z_j)p(z_j),$$

where $q(z_j)$ denotes the (positive or negative) quantity held of a security that pays a unit of consumption upon realization of income z_j, and is traded at a price $p(z_j)$, $j = 1, \ldots, n$ that is not influenced by each individual's choices.

The notation can accommodate non-contingent assets in a multiple-period economy where consumption and income are indexed by time as well as random realizations. For example, if for some $j = N$ it is the case that $p(z_N) = (1 + r)^{-1} r$ and $f(z_N|e) = 1/\beta$, then the return of this asset does not depend on the income realization. The individual can then allocate some resources to first-period consumption (when utility is weighted by an inverse discount factor rather than a probability, and a non-random endowment may be available) rather than to second-period consumption.

The first-order approach is valid if the objective function (1) is concave. If effort decreases welfare at a non-decreasing rate,

[A1] $v'(e) > 0,\ v''(e) \geq 0\ \forall\, e > 0,$

and the utility function $u(c)$ is strictly concave in consumption,

[A2] $u'(c) > 0,\ u''(c) < 0\ \forall\, c,$

it is straightforward to establish concavity when there is no moral hazard:

Remark 1. If $f(z_i|e) = \mathbf{f}(z_i)$ for all i, then assumptions [**A1**] and [**A2**] suffice to ensure concavity of the objective function (1) in effort e and the quantities $\{q(z_i)\}_{i=1}^{n}$.

Concavity, of course, follows immediately from the fact that the objective function is a linear combination of concave transformations of linear functions. It is however useful in what follows to refer to a more detailed proof (in the Appendix) that explicitly proves concavity exploiting block-diagonality in effort and security quantities of the objective function's Hessian when probabilities are exogenously given, and showing that the Hessian of a function that maps to the real line a linear combination of variables is negative definite when that function is concave.

3. Establishing concavity under moral hazard

Remark 1's detailed derivation of a fairly obvious result highlights the problems arising when probabilities depend on effort. Even when effort costs are linearly separable in the objective function, if $f(z_i|e)$ depends on e it does not appear possible to replicate the steps of Remark 1's proof and characterize in full generality how the form of the model's primitive functions bears on negative definiteness of the Hessian of (1). Doing so would be exceedingly complicated when not only moral hazard prevents the objective function from being a linear combination of concave functions, but also hidden asset positions appear among the function's arguments. While Jewitt (1988) shows how functional restrictions can establish concavity with respect to effort choices in moral hazard problems without hidden assets, such a direct approach is very cumbersome when derivatives with respect to asset quantities also appear in the Hessian of a multivariate objective function.

Adopting the approach of Rogerson (1985) instead, one may rewrite (1) as

$$\begin{aligned} & -v(e) + \sum_{i=1}^{n} u(c(z_i))f(z_i|e) \\ & = -v(e) + u(c(z_n)) + \sum_{i=1}^{n-1} f(z_i|e)\left(u(c(z_i)) - u(c(z_n))\right) \\ & = -v(e) + u(c(z_n)) - \sum_{i=1}^{n-1} F(z_i|e)(u(c(z_{i+1})) - u(c(z_i))), \qquad (2) \end{aligned}$$

where F is the cumulative distribution function. It follows from [**A1**], [**A2**], and linearity of $c(z_n)$ in $\{q(z_i)\}_{i=1}^{n}$ that $-v(e) + u(c(z_n))$ is concave in e and $\{q(z_i)\}_{i=1}^{n}$. Thus, it would suffice to establish concavity of the weighted sum of utility differences that appear in (2).

To characterize a summation of products of probability and utility functions, it is potentially useful to make assumptions about the form of these functions. As in Ábrahám et al. (2011) and Bertola and Koeniger (2013), a promising restriction for the distribution function is

[A3] $F(z_i|e)$ is log-convex in effort.

This is a stronger restriction than the convexity assumption that in Rogerson (1985) suffices to prove concavity of the objective function in the absence of hidden assets when [**A2**] holds and the likelihood ratio of $f(z_i|e)$ is monotone.[3] Should restrictions on the form of the utility function that are stronger than [**A2**] ensure log-convexity of $u(c(z_{i+1})) - u(c(z_i))$, when consumption levels are influenced by hidden asset positions, then [**A3**] implies that the terms in the last summation above are convex (because log-convexity is preserved in multiplication and implies convexity), and concave when entering the objective function with a minus sign.

1 In this setting a non-exclusive market can be active for trade in securities contingent on idiosyncratic realizations (the sale of securities is non-exclusive since agents can purchase securities from more than one provider). If instead observable outcomes are a deterministic function of effort choices based on privately observed ability realizations, as in the hidden-information economies analyzed by Mirrlees (1971) or Cole and Kocherlakota (2001), private information rules out trade in non-exclusive contingent securities (Golosov and Tsyvinski, 2007, Appendix A).

2 A non-negativity constraint on consumption is not imposed, or is not binding because the marginal utility of consumption diverges to infinity at zero.

3 The condition of a monotone likelihood ratio requires that $(\partial f(z_i|e)/\partial e)\ /f(z_i|e)$ be non-decreasing in z_i which has the natural interpretation that more effort increases output on average (see Rogerson, 1985, and his references).