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Optimal investment policy with fixed adjustment costs and complete irreversibility

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HIGHLIGHTS

- We model investment decisions with a fixed adjustment cost not proportional to existing capital.
- The optimal policy is of the generalized (S, s) form.
- In agreement with the empirical evidence, as the firm size increases, investment becomes less lumpy.

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ABSTRACT

We develop and characterize analytically an investment model in discrete time with a fixed adjustment cost not proportional to existing capital and complete irreversibility that reproduces the lumpiness of investment at the micro-level. In agreement with the empirical evidence, as a firm size increases, investment becomes less lumpy. The optimal policy is of the generalized (S, s) form.

1. Introduction

Investment is lumpy at the plant level with long periods of inactivity punctuated by infrequent and large adjustments. And, smaller plants have lumpier investment patterns: both the probability of a spike and the share of total investment represented by spikes are decreasing in establishment size.[1]

We consider a model of lumpy investment that has received a lot of attention in both applied and theoretical works.[2] Existing theoretical models cannot account for size-dependent lumpiness. Fixed investment costs are proportional to the existing capital stock in Caballero and Leahy (1996), proportional to current profitability in Abel and Eberly (1998) and proportional to current output in Caballero and Engel (1999). Such fixed costs do not become irrelevant as the firm grows but imply that the firm size does not matter for investment dynamics.

We extend this theoretical literature by considering a fixed adjustment cost not proportional to the existing capital.[3] We provide a characterization of the optimal policy and the value function under general assumptions on uncertainty and technology. The

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[1] These findings have been reported in a number of datasets for the US (Doms and Dunne, 1998), Latin America (Gelos and Isgut, 2001), Europe (Nilsen and Schiantarelli, 2003; Nilsen et al., 2007; Bachmann and Bayer, 2014), Africa and Asia (Bond et al., 2007).

[2] See Abel and Eberly (1994), Dixit and Pindyck (1994), Caballero and Engel (1999), Khan and Thomas (2008), Stokey (2008), Bachmann et al. (2013), and Bachmann and Bayer (2014). Fixed adjustment costs are recognized to be important in many other settings such as entry into the labor market (Cogan, 1981), labor demand (Cooper et al., 2004) or durable goods consumption (Grossman and Laroque, 1990).

[3] Recent quantitative work has used a similar specification but rely on numerical methods to characterize the solution. See Khan and Thomas (2008), Bachmann et al. (2013), Elsby and Michaels (2014) and Bachmann and Bayer (2014).

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main difficulty created by the fixed cost is that the value function is no longer concave and hence standard arguments (see Stokey et al., 1989) need to be extended. Our contribution is threefold. First, we show that the concept of k-concavity introduced by Scarf (1960) can be applied to an investment model to derive the optimal policy which is of the generalized (S, s) form.[4] Other existing proofs of the optimality of (S, s) policies in investment models use a continuous-time setting.[5] Yet, discrete time allows us to characterize the optimal policy in a rigorous and parsimonious way whereas continuous-time models can be impeded by measure-theoretic issues. In addition, discrete-time models are familiar to a broader audience since they are the basis of most modern macroeconomics and empirical work. Second, we show that the closeness of the space of k-concave functions follows almost directly from the property that k-concavity is preserved after maximization of a k-concave function minus a fixed cost k. Last, a substantive contribution is to show that under plausibly calibrated values the model delivers more lumpiness for smaller plants, a robust feature of plant-level data, while in existing theoretical models lumpiness is not systematically related to the plant size.

An important restriction we impose throughout the paper is complete irreversibility. Relaxing this assumption creates technical difficulties: the optimal policy is no longer guaranteed to be of the (S, s) form and the value function is not k-concave.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the optimal decision rule and its properties. Section 4 presents the implications of the model for the lumpiness of investment by firm size. Section 5 concludes.

2. The model

2.1. Assumptions

Time t is discrete and indexed by t. At each period, the plant's manager decides to invest or not ($i_t \geq 0$) over an infinite time horizon. She is risk-neutral and discounts future profits at a constant rate $\beta \in (0, 1)$. Her decision depends on the level of capital inherited from the previous period $k_t \in \mathcal{K}$ and the plant's profitability $A_t \in \mathcal{A}$. The level of capital at the start of the next period, $t+1$, is: $k_{t+1} = (1-\delta)k_t + i_t$ where δ is a positive depreciation rate. The one-period profit function is:

$$\pi(A, k, i) = R(A, k) - C(i) \tag{2.1}$$

$R(A, k)$ represents reduced-form revenue and incorporates the optimal choice of flexible factors. $C(i)$ is the adjustment cost function. Profitability A follows an exogenous and stationary Markov process. New values of profitability are drawn from a Markov transition function $Z : \mathbf{A} \times \mathcal{A} \longrightarrow [0, 1]$ where $(\mathcal{A}, \mathbf{A})$ is a measurable space. $\mathcal{A}$ is a Borel set in $\mathbb{R}_+$, with its Borel subsets $\mathbf{A}$.

Assumption 1. $\mathcal{A}$ and $\mathcal{K} \equiv \left[0, \bar{k}\right]$ are compact sets in $\mathbb{R}_+$ with $\bar{k} < \infty$.

Assumption 2. Z has the Feller property.

Assumption 3. (i) R is continuous on $\mathcal{A} \times \mathcal{K}$ and (ii) $R(A, \cdot)$ is concave for each $A \in \mathcal{A}$.

We consider the following specification for adjustment cost.

$$C(i) = \begin{cases} F + pi & \text{if } i > 0 \\ 0 & \text{if } i = 0 \\ \infty & \text{if } i < 0 \end{cases} \tag{2.2}$$

where F and p represent, respectively, fixed and linear adjustment costs. The fixed cost creates a discontinuity in $C(\cdot)$ at 0: $C(0) = 0$ while $\lim_{i \to 0^+} C(i) = F > 0$.

Fixed investment costs are proportional to the existing capital stock Fk in Caballero and Leahy (1996), proportional to current profitability FA in Abel and Eberly (1998) and proportional to current output $FR(A, k)$ in Caballero and Engel (1999). Our formulation leads to more lumpiness for smaller plants as shown in Section 4.

2.2. The dynamic programming problem

Given the law of motion of capital $k_{t+1} = (1-\delta)k_t + i_t$, $i_t \geq 0$ and the current state (A_t, k_t), a manager chooses the sequence of investment $\{i_j\}_{j=t}^{\infty}$ to maximize the present discounted value of current and future profits:

$$V^*(A_t, k_t) = \sup_{\{i_j\}_{j=t}^{\infty}} E\left[\sum_{j=t}^{\infty} \beta^{j-t} \pi(A_j, k_j, i_j) | k_t, A_t\right] \tag{2.3}$$

where $V^\star$ is the supremum function. The value function $V(A, k)$ is given by a solution to Bellman's equation:

$$V(A, k) = R(A, k) + \sup_{k(1-\delta) \leq k' \leq \bar{k}} \left[-C(k' - k(1-\delta)) + \beta \int_{\mathcal{A}} V(A', k') Z(dA', A)\right]. \tag{2.4}$$

The results from Stokey et al. (1989) relating the sequence problem in Eq. (2.3) to the functional equation (2.4) apply and we can focus on the Bellman equation and its solution.[6] The following sets are defined under the usual sup-norm $\|\cdot\|$. Let $\mathcal{B}$ be the set of bounded functions $V : \mathcal{A} \times \mathcal{K} \longrightarrow \mathbb{R}$. Let $\mathcal{C}$ be the set of bounded and continuous functions $V : \mathcal{A} \times \mathcal{K} \longrightarrow \mathbb{R}$. Let T be the Bellman operator in Eq. (2.4) with $V = T(V)$. Next proposition shows that the value function is continuous despite the discontinuity in the one-period profit function.

Proposition 4. *Under Assumptions 1–3, V^* is jointly continuous in (A, k).*

Proof. Applying Theorem 4.6 in Stokey et al. (1989) to bounded functions gives that V^* is the unique function in $\mathcal{B}$ that satisfies the Bellman Eq. (2.4). The value function can be re-written as:

$$V(A, k) = R(A, k) + \max\left\{\sup_{k(1-\delta) \leq k' \leq \bar{k}} \left\{\beta \int_{\mathcal{A}} V(A', k') Z(dA', A) - p(k' - k(1-\delta)) - F\right\}, \beta \int_{\mathcal{A}} V\left(A', k(1-\delta)\right) Z(dA', A)\right\}.$$

[4] Our setting differs from Scarf (1960) for several reasons. Notably, we model investment decisions of a profit maximizing firm, the horizon is infinite and profitability follows a Markov process.

[5] See Dixit and Pindyck (1994) and Stokey (2008) for reviews of continuous-time methods and applications.

[6] Theorems 9.2 and 9.4 in Section 9.1 of Stokey et al. (1989) hold under the assumptions that (a) π is bounded above and (b) the correspondence $\left\{k' \in \mathcal{K} : k' \geq k(1-\delta)\right\}$ is non-empty valued and has a measurable selection and its graph is measurable. It is immediate that under Assumptions 1–3 both (a) and (b) are satisfied.